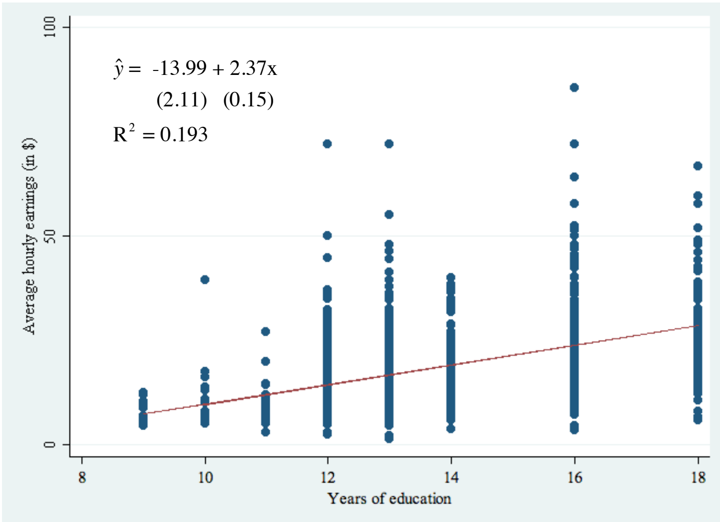
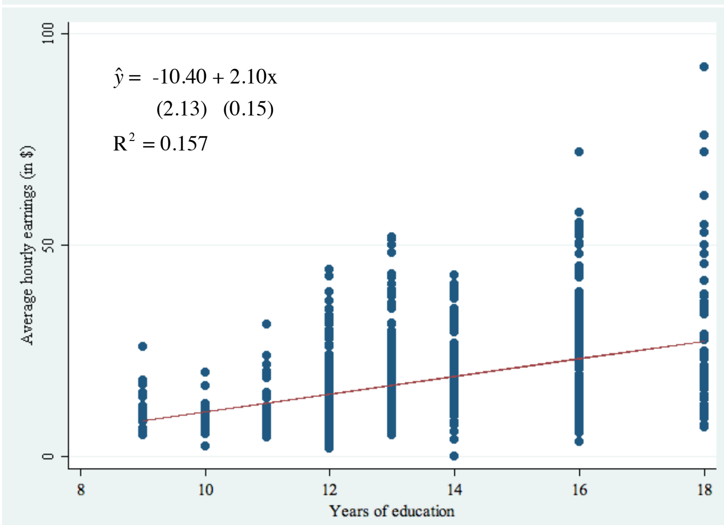
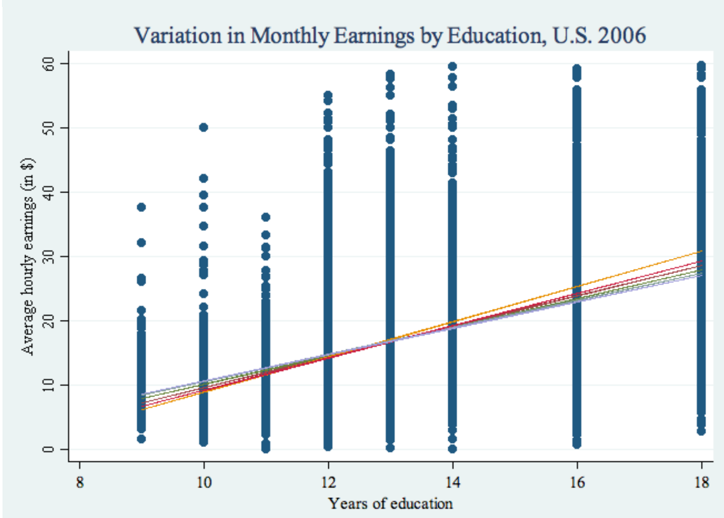
Illustrating that estimators are random variables:
We draw 13 independent samples of 1000 observations from the population of US workers







Estimated slopes with 13 samples
2.291
2.567
2.311
2.333
2.183
2.292
2.081
2.109
2.035
2.211
2.385
2.466
2.463
2.287

1. Repeating the same random sampling of 1000 observations gives different estimates, but if you were to average them up, you would find the true value, because $E\left(\hat{\beta}\right) = \beta$

2. Increasing sample size increases the precision of the estimate, because $\mathrm{var}\left(\hat{\beta}_1\right)=\dfrac{\mathrm{var}(u)}{\mathrm{SST}_x}$

```
. reg wage educ

      Source |       SS       df       MS              Number of obs =     400
-------------+------------------------------           F(  1,   398) =   69.64
       Model |  7947.97607     1  7947.97607           Prob > F      =  0.0000
    Residual |  45425.1083   398  114.133438           R-squared     =  0.1489
-------------+------------------------------           Adj R-squared =  0.1468
       Total |  53373.0843   399  133.767129           Root MSE      =  10.683

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   2.166719   .2596455     8.34   0.000     1.656271    2.677168
       _cons |  -11.09661   3.579697    -3.10   0.002    -18.13409   -4.059135
------------------------------------------------------------------------------
```

```
. reg wage educ

      Source |       SS       df       MS              Number of obs =    2000
-------------+------------------------------           F(  1,  1998) =  333.22
       Model |  41122.3613     1  41122.3613           Prob > F      =  0.0000
    Residual |  246572.252  1998  123.409535           R-squared     =  0.1429
-------------+------------------------------           Adj R-squared =  0.1425
       Total |  287694.613  1999  143.919266           Root MSE      =  11.109

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   2.203853   .1207308    18.25   0.000     1.967082    2.440625
       _cons |  -11.93304   1.661577    -7.18   0.000    -15.19164   -8.674433
------------------------------------------------------------------------------
```

```
. reg wage educ

      Source |       SS       df       MS              Number of obs =    4000
-------------+------------------------------           F(  1,  3998) =  728.91
       Model |  85546.3393     1  85546.3393           Prob > F      =  0.0000
    Residual |  469213.909  3998  117.362158           R-squared     =  0.1542
-------------+------------------------------           Adj R-squared =  0.1540
       Total |  554760.248  3999  138.724743           Root MSE      =  10.833

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   2.221041   .0822659    27.00   0.000     2.059754    2.382328
       _cons |  -12.02081   1.133032   -10.61   0.000    -14.24218   -9.799433
------------------------------------------------------------------------------
```

Back to our sample of 2000 workers

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. sum wage educ

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
        educ |      2000      13.633      2.0877          9         18
        wage |      2000    18.34701    11.49495         .7   82.42857

. reg wage educ

      Source |       SS       df       MS              Number of obs =    2000
-------------+------------------------------           F(  1,  1998) =  376.94
       Model |  41922.0349     1  41922.0349           Prob > F      =  0.0000
    Residual |  222213.443  1998  111.217939           R-squared     =  0.1587
-------------+------------------------------           Adj R-squared =  0.1583
       Total |  264135.478  1999  132.133806           Root MSE      =  10.546

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   2.193546    .112983    19.41   0.000     1.971969    2.415123
       _cons |   -11.5576   1.558244    -7.42   0.000    -14.61355   -8.501649
------------------------------------------------------------------------------
```

Calculating $\mathrm{se}\left(\hat{\beta}_1\right)$

The unknown true values for the variance and standard deviation of the random variable $\hat{\beta}_1$ are:

$$\mathrm{var}\left(\hat{\beta}_1\right)=\frac{\sigma_u^2}{\mathrm{SST}_x} \quad \text{and} \quad \mathrm{sd}\left(\hat{\beta}_1\right)=\frac{\sigma_u}{\sqrt{\mathrm{SST}_x}}$$

Estimation are obtained by replacing σ_u^2 by an estimation computed from the residuals:

Compute $\hat{\sigma}_u^2=\frac{\sum_i \hat{u}_i^2}{n-2}=\frac{\mathrm{SSR}}{n-2}=\frac{222213.44}{1998}=111.22$

Compute $\mathrm{SST}_x=(2.088)^2\,1999=8715.13$

Compute $\widehat{\mathrm{var}}\left(\hat{\beta}_1\right)=\frac{\hat{\sigma}_u^2}{\mathrm{SST}_x}=\frac{111.22}{8715.13}=0.01276$ $\qquad$ $\mathrm{se}\left(\hat{\beta}_1\right)=\sqrt{0.01276}=0.1130$