Climate Jumps, Fat Tails and Non-linear Carbon Uptake

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Motivation

- Recent interest in “fat tails” in distribution over temperatures
- one motivation: survey of published scientific work (Weitzman)
- subjective appraisals of key parameters, unclear these are independent

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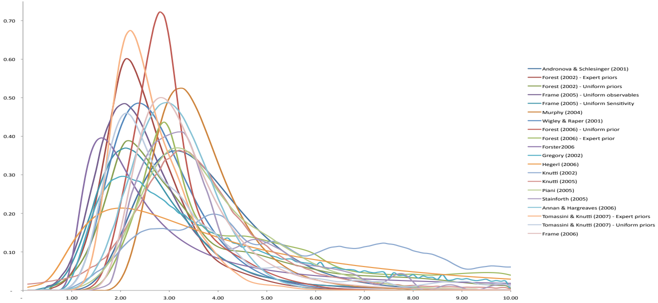

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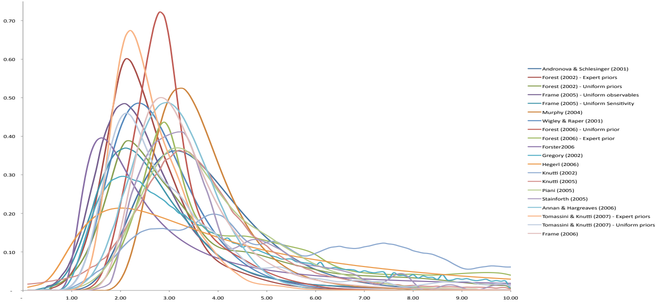

- can we say something using current climate data?

Motivation

Current economic literature typically assumes:

- damages based on carbon stock not temperature
- exponential decay of carbon stock (linear uptake)
- stylized [quadratic] representation of link between climate and damages
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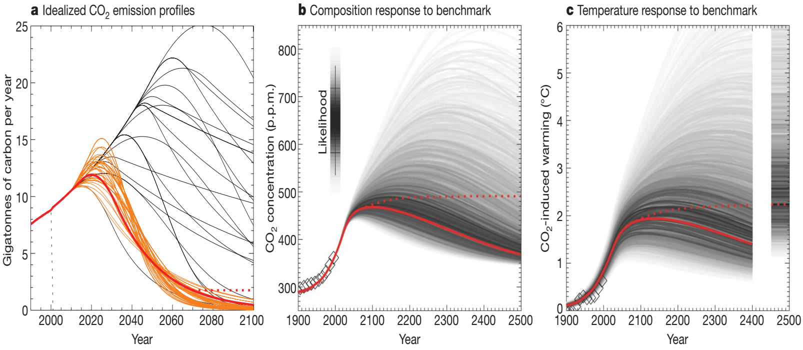
Combined, these assumptions imply downward bias in social cost of carbon

- relating damages to carbon stocks only sensible if direct relation
- climate scientists recently observed some carbon ($\approx 20\%$) stays in atmosphere virtually indefinitely $\Rightarrow$ concave (non linear) uptake
- quadratic damages $\Rightarrow$ focus on mean & variance
  - higher-order moments not relevant

carbon stock vs. temperature

- temperature changes linked to carbon stock
- carbon stock changes linked to emissions
- suggests 2^{nd} order relation between temperature and accumulated emissions



carbon stock dynamics

- relating damages to carbon stocks only sensible if direct relation
- Physicists typically assume “three box model”
- 3 state variables, related to different time frames
  - C_3 reflects ‘long term equilibrium ’ stock
  - C_2 reflects medium term variations around C_3
  - C_1 represents shorter term variations
- importantly, gases depreciate from C_3 so slowly as to be negligible
- implies some carbon ($\approx 20\%$) stays in atmosphere virtually indefinitely
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Three box model

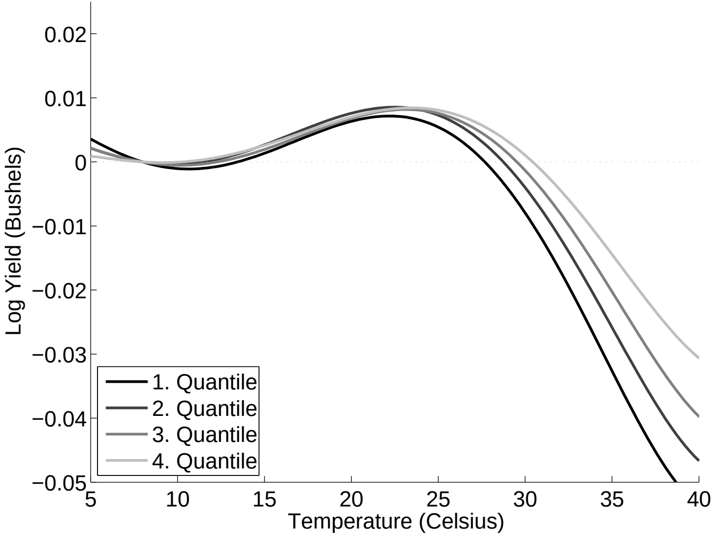
$$\begin{aligned}\dot{C}_2 &= a_0 E - b_2 C_2 \\ \dot{C}_3 &= b_3 C_3\end{aligned}$$

- concave relation in $\dot{C}$... approximate with quadratic decay?
- analogous to the logistic growth component in modern fisheries models

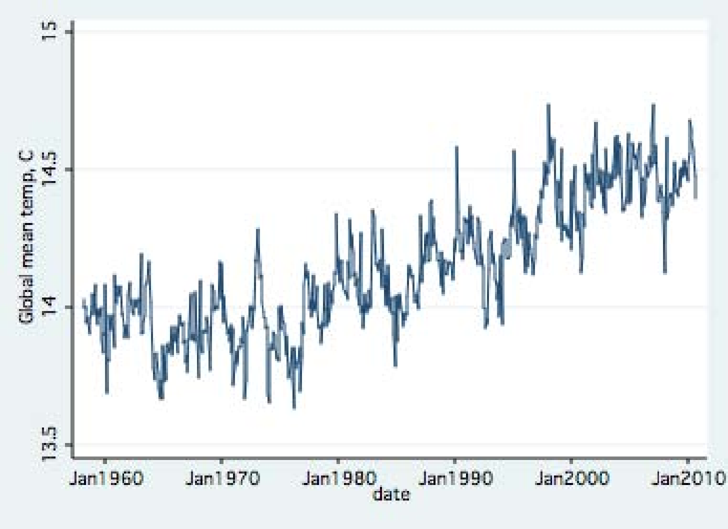
$$\dot{C} = a_1 E - U(C) = a_1 E - C(b_0 - b_1 C)$$

- $U(C)$ is "uptake"
- interpretation of coefficient on C^2: "carrying capacity"
- maximum ability of sinks to uptake carbon
  - oceans
  - forest stocks

Linear marginal damage?



stochastic temperatures?



Control model

- PDV of payoffs at time $t = [\pi(E) - D(T)]e^{-\rho t}$,
  - π: net benefits from unabated emissions [GDP net of seq'n, abatem't costs]
  - D: temperature-related damages
  - ρ: discount rate
- $\pi'(E) > 0$ for small $E, \pi''(E) < 0$
- iso-elastic form $\pi(E) = AE^{\theta+1}$ has received significant attention
  - elasticity $\theta = \frac{\pi'(E)}{E\pi''(E)}$
- define Current-value Hamiltonian

$$\mathbf{H} = \pi(E) - D(T) + \mu[a_1 E - C(b_0 - b_1 C)] + \nu[\alpha \ln(\frac{C}{C_0}) - \beta T]$$

  - μ is co-state variable (shadow value) associated with state variable C
  - ν is co-state variable (shadow value) associated with state variable T

Maximum principle

Necessary conditions for solution:

$$
\begin{aligned}
0 &= \pi'(E^*) + a_1\mu \\
\dot{\mu} &= \rho\mu - \partial\mathbf{H}/\partial C = (\rho + b_0)\mu - 2b_1 C\mu - \frac{\alpha}{C}\nu \\
\dot{\nu} &= \rho\nu - \partial\mathbf{H}/\partial T = (\rho + \beta)\nu + D'(T)
\end{aligned}
$$

Time-differentiate equation first condition to obtain

$0 = \pi''(E^*)\dot{E}^* + a_1\dot{\mu} = \pi''(E^*)\dot{E}^* + a_1[(\rho + b_0)\mu - 2b_1 C\mu - \frac{\alpha}{C}\nu]$, or

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$$\dot{E}^* = (\rho + b_0 - 2b_1 C)\frac{\pi'(E^*)}{\pi''(E^*)} - \frac{a_1\alpha}{\pi''(E^*)C}\nu \leftrightarrow$$

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$$\dot{E}^* = (\rho + b_0 - 2b_1 C)\frac{\pi'(E^*)}{\pi''(E^*)} - \frac{a_1\alpha}{\pi''(E^*)C}\nu \leftrightarrow$$

$$\frac{\dot{E}^*}{E^*} = U'(C)\theta - \frac{a_1}{E^*\pi''(E^*)}\frac{\alpha\nu}{C}$$

Simpler model

Consider 'conventional' assumption $T(t) = \phi(C(t))$

- damages are then $d(C) \equiv D(\phi(C))$
- state equation on T becomes irrelevant to the dynamic optimization problem
- optimality condition for E is as above
- equation of motion for the (lone remaining) co-state variable is

$$\dot{\mu} = (\rho + b_0)\mu - 2b_1 C\mu + d'(C)$$

- differential equation governing the path of optimal emissions becomes

$$\frac{\dot{E}}{E} = U'(C)\theta - \frac{a_1}{E\pi''(E)} d'(C)$$

- replace the component $\frac{\alpha\nu}{C}$ with $d'(C), E^*$ with E

Data

- Temperature: monthly global mean temperature, Centigrade
- Carbon stocks: monthly readings at Mauna Loa, ppm
- March 1958 - August 2011
- Global emissions: annual observations, 1958 - 2007
  - ‘conventional’ emissions
  - emissions related to land use change
- data sources: NOAA, CDIAC, EIA

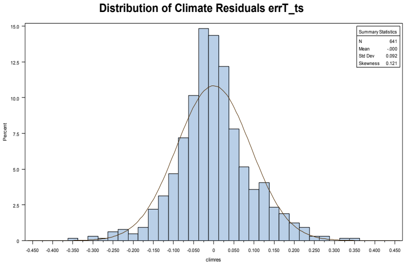
Stochastic temperatures, cont.

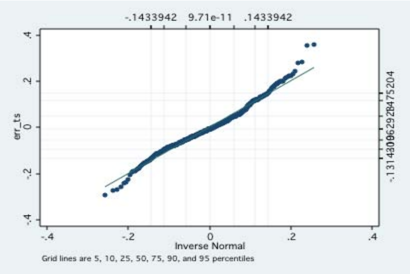
- now represent temperature state equation as stochastic
- how to model? GBM? jump process?
- regress $\ln(C_{t-1})$, T_{t-1} on $T_t - T_{t-1}$ $(= \Delta T)$
  - alternative 1: if C drives T then ΔC drives ΔT
  - alternative 2: perhaps outliers
  - alternative 3: endogeneity in carbon?

variable	Regression 1	Regression 2	Regression 3	Regression 4	Regression 5
T_{t-1}	-.1634**	-.1639**	-.0403**	-.1587**	-.1647**
	(.0217)	(.0219)	(.0114)	(.0196)	(.0223)
$\ln(C_t)$	.5167**	.5167**	—	.5058**	.5361**
	(.0787)	(.0789)	—	(.0711)	(.0834)
$C_t - C_{t-1}$	—-	.0029	.0308*	—	—
	—	(.0167)	(.0135)	—	—
D_{out0}	—	—	—	-.0957**	—
	—	—	—	(.0019)	—
D_{out1}	—	—	—	.1075**	—
	—	—	—	(.0019)	—
constant	-.9647**	-.9560	.7767**	9699**	-1.0943**
	(.3368)	(.3394)	(.2198)	(.3038)	(.3604)
R-squared	.082	.082	.020	.256	.082

Dependent variable: $T_t - T_{t-1}$
number of observations = 640

Normally distributed residuals?





- kurtosis = 4.285
- probability this does not differ from 3 (Normal) $< .01$

Jumps?

- allow for probability of jump = λ
- distribution of jump sizes
- mean θ, variance δ^2
- means and variance of residuals when no jump obtains: μ, σ^2
- similar qualitative results to kurtosis test

coefficient	estimate	std. err.	restricted estimate	restricted std.err.
μ	-.1209	.043	-.0001	.040
σ	.4196**	.049	.9206**	.026
λ	.5816	.069	—	—
θ	.2076*	.089	—	—
δ	1.0661**	.060	—	—

Chi-squared test statistic = 54.87
p-value $< .0001$
*: significant at 5% level
**: significant at 1% level

Carbon Decay

- annual data on forest stocks 1990-2008
- use land use data to predict forest stocks going back to 1958
  - use this synthetic data to create a proxy D for deforestation
  - D represented relative to 1958 (% of land deforested)
- use changes in CO2 stock, total emissions to construct variable representing ‘uptake’
- regress uptake on C, C^2 and perhaps interactions with forest stocks
  - H_0 : coefficients on C^2 are statistically unimportant

Uptake results

Regression results: carbon uptake analysis

variable	coefficient	std. err.	t-stat
C	1.2460	.4369	2.85
C^2	-.00851	.00370	2.30
FC	-1.0483	.4953	2.12
FC^2	.00314	.00146	2.16
constant	536.95	290.24	1.85

R^2 = .803

Durbin-Watson stat = 1.909

F-stat on H_0: 5.465 (1% critical value = 5.149)

Theoretical considerations

DP approach: solve for V(C,T) using Fundamental eq'n of optimality

Deterministic modeling approach:

$$\mathbf{max}_{x_t} \left\{ \pi_t e^{-rt} + \dot{C}\frac{\partial V}{\partial C} + \dot{T}\frac{\partial V}{\partial T} \right\} = \rho V.$$

Stochastic variant:

$$\mathbf{max}_{x_t} \left\{ \pi_t e^{-rt} + \dot{C}\frac{\partial V}{\partial C} + \frac{1}{dt}E\left[dT\right]\frac{\partial V}{\partial T} \right\} = \rho V.$$

Expand Ito operator:

$$\frac{1}{dt}E\left[dT\right] = \underbrace{\alpha \ln\left(C/C_0\right) - \beta T}_{\text{deterministic ingredients}} + \underbrace{\chi(\mu, \sigma^2, \lambda, \theta, \delta)}_{\text{stochastic ingredients}}$$

Concluding thoughts

- Important to shift focus from carbon stock to temperature
  - leads to more complicated, subtler, effects
- some evidence of relatively fat tails in residuals associated with temperature changes
  - suggests fatter-tailed distribution than Brownian motion
  - possible role for unanticipated rapid changes (*jumps*)
- evidence of non-linear decay in carbon stocks
  - important in both ocean and forest sinks
  - forest sinks absorb less rapidly, non-linear effect enters less rapidly