The Political Economy of Environmental Policy with Overlapping Generations*

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Abstract

A two-sector OLG model illuminates previously unexamined intergenerational effects of a tax that protects an environmental stock. A traded asset capitalizes the economic returns to future tax-induced environmental improvements, benefiting the current asset owners, the old generation. Absent a transfer, the tax harms the young generation by decreasing their real wage. Future generations benefit from the tax-induced improvement in environmental stock. The principal intergenerational conflict arising from public policy is between generations alive at the time society imposes the policy, not between generations alive at different times. A Pareto-improving policy can be implemented under various political economy settings.

Keywords: Open-access resource, two-sector overlapping generations, resource tax, generational conflict, climate policy, dynamic bargaining, Markov perfection

JEL, classification numbers: E24, H23, Q20, Q52, Q54

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1 Introduction

Most analyses of stock-related environmental problems use assumptions that imply that people alive today must make sacrifices in order to preserve consumption opportunities for those alive in the future. This analysis, in focusing on the conflict between agents who live at different points in time, tends to ignore the conflict between different types of agents alive when the policy is first implemented. An overlapping generations (OLG) model with endogenous asset prices turns this conventional view on its head: all generations may benefit from environmental policy, provided that the winners alive today compensate those who would, absent compensation, be harmed by the policy.

Climate policy models with endogenous capital, such as DICE (Nordhaus 2008), use a one-commodity setting in which output can be either consumed or invested. When investment is positive – as is always the case in equilibrium – the normalization that sets the commodity price to 1 implies that the asset price is fixed, also at 1. In these models, climate policy affects the trajectory of an environmental stock, such as temperature, which in turn affects the future productivity of capital and thereby affects current investment. In this setting, the trajectory of capital is endogenous but the price of capital is fixed. The fixed asset price means that these models exclude a potentially important mechanism whereby policy-induced changes in future productivity effect the level and distribution of welfare.

For the purpose of examining the role of asset prices, we study a model that reverses these assumptions: there is a fixed or exogenously changing stock of capital and no depreciation, forcing the price of capital to be endogenous and responsive to policy-induced changes in future productivity. For given environmental stocks, stricter environmental policy reduces current real aggregate income, exactly as in previous models. Stricter policy also lowers the current real wage and rental rate of capital; in that respect the welfare effect of policy is symmetric across factors of production. However, by increasing future productivity via improved future environmental stocks (relative to Business as Usual, or BAU), the stricter policy increases the *price* (as distinct from *rental rate*) of the asset. In the particular model that we study, the higher price more than offsets the lower current rental rate, and environmental policy increases the welfare of current owners of capital. Although policy has symmetric effects on the current real wage and rental rate, there is a basic asymmetry between capital and labor: the price of the former reflects future productivity, whereas the price of the latter depends only on current productivity since agents provide labor only in the first period of their life. This difference drives the welfare effects that we examine.

In our OLG setting, agents live for two periods; we consider an environmental tax. The current old generation owns capital, which it sells to young agents. Because the policy lowers current aggregate real income and increases the welfare of old agents, it necessarily decreases the first period utility of young agents. Young agents might benefit from the policy-induced environmental improvement in the second period of their life. In circumstances that are relevant to climate change this offsetting improvement does not compensate the young for the loss of first period utility. Therefore, absent transfers, climate policy increases lifetime welfare of the old asset-rich and lowers the lifetime welfare of the young asset-poor. However, the old generation can retain all of the benefits of the higher asset values and compensate the young merely by giving them a larger share of the revenue from the environmental tax, compared to the share that future young generations will obtain.

In this way, the old rich pay the young poor to accept stricter climate policy. They make this transfer not because of a moral imperative, but because it is in their interest to do so: absent the transfer, the young have no reason to agree to implement the policy. There is an obvious (albeit imperfect) correspondence between the old rich and the young poor in our model, and the older developed countries and younger developing countries in the real world. The model thus provides a motivation, based on self-interest, for transfers from the rich developed to the poor developing countries, in order to induce the latter to implement climate policy.

A richer model would nest our model and Ramsey-type models by allowing endogenous investment and depreciation, together with convex adjustment costs in investment. The convexity of investment costs makes the price of capital endogenous, even though the commodity price is fixed by normalization. This more general model approximates the Ramsey-type models if investment costs are nearly linear, *or* if capital depreciates quickly relative to the speed of policy-induced environmental and productivity changes. If investment costs are extremely convex *and* capital depreciates slowly relative to policy-induced productivity changes, the richer model approximates the special model that we study. We do not claim that our model is more realistic, or a better guide to policy, compared to Ramsey-type models – merely that it offers a different perspective on climate policy, and is a useful stepping stone to the general model. The general model relies on numerical methods, whereas we obtain analytic results in the simple setting. Details of the general model are beyond the scope of this paper, but provisional numerical analysis confirms that the results of the simple model (the subject of this paper) are not "knife-edge". As in the previous literature where capital is important, we aggregate all types of capital into a single stock – an assumption that we regard as relatively innocuous.

We consider two types of policy settings. In the first, we obtain analytic results, summarized above, for arbitrary perturbations from BAU. We then use numerical methods to study the equilibrium in a dynamic bargaining game. In each period of this game, the current old and young generations pick a current tax to maximize their aggregate lifetime welfare. They understand that this tax affects the evolution of the environmental stock, which influences the equilibrium tax chosen in the next period. Future taxes affect future environmental stocks, and thus affect the future productivity of capital; that in turn affects the current asset price, and thereby affects the welfare of current generations.

We need specific functional forms for the numerical analysis. The functional assumptions also affect the tractability, but not the intuition, of the qualitative analysis. We therefore use a particular two-sector general equilibrium model, one that leads to simple expressions for the equilibrium real wage, real rental rate, and aggregate real income as functions of the current tax and environmental stock. As noted above, environmental policy lowers current aggregate real income (exactly as in Ramsey-type models) for predetermined current environmental stock. In addition, it is symmetric with respect to factors insofar as it lowers both the real wage and the real rental rate. The asymmetry between factors arises only because the *price* of capital reflects future productivity changes, whereas the wage (like the rental rate) depend only on current productivity.

2 Literature review

The literature that examines environmental policy in OLG models has neglected the particular role of asset prices that we emphasize. Kemp and van Long (1979) and Mourmouras (1991) are among the first to use the OLG framework of Samuelson (1958) and Diamond (1965) to assess the economics of renewable resources. Mourmouras (1993) demonstrates that a social planner can implement welfare-improving conservation measures in a model with environmental externalities and capital accumulation. The emphasis of these policies is to implement non-decreasing ("sustainable") consumption paths. Howarth (1991, 1996), Howarth and Norgaard (1990, 1992), and Krautkraemer and Batina (1999) analyze welfare aspects of sustainable consumption paths in OLG models. John et al. (1995) discuss the steady state inefficiencies due to intergenerational disconnectedness in the presence of private goods with negative externalities; John and Pecchenino (1994) consider the transitional dynamics in this setting. Marini and Scaramozzino (1995) analyze the intertemporal effects of environmental externalities and optimal, time-consistent fiscal policy in continuous time. These contributions recognize that environmental policy affects different generations unevenly because costs are immediate but benefits arise in the future.

Bovenberg and Heijdra (1998, 2002) and Heijdra et al. (2006) show that the issuance of public debt can be used to achieve intergenerational transfers, leading to Pareto improvements; they examine the difference in distributional impacts of profit, wage, and lump-sum taxes. Our contribution emphasizes the role of asset price effects and shows that Pareto-improving tax policy can be implemented and sustained through an endogenous political process. In particular, an environmental tax can improve current generations' welfare even in the absence of a government that uses bonds to distribute income across generations. We use a dynamic general equilibrium model that, apart from the OLG structure, is similar to that of Copeland and Taylor (2009). It is close to that of Koskela et al. (2002) in its OLG structure; but differs by separating conventional capital and the renewable resource into different sectors and by allowing for open-access in the latter. Galor (1992) discusses existence and stability properties of a general two-sector OLG model. Farmer and Wendner (2003) extend Galor's insights to models with heterogeneous capital. Our assumption of a fixed capital stock reduces much of the complexity of their models and creates linkages to the asset price model of Lucas (1978). Many papers use the fact that asset prices depend on adjustment costs, without, however developing the idea that asset prices can provide an incentive for the current generation to improve the welfare of future generations (Huberman 1984, Huffman 1985 and 1986, and Labadie 1986).

The insights of our model are applicable where there are imperfect property rights to an endogenously changing natural stock, as is the case with the climate. Most models used to study climate policy include assumptions that imply that meaningful policy requires a reduction in current consumption and current utility. Howarth (1998), Rasmussen (2003) and Leach (2008) present numerical estimates of the welfare impacts of climate policy in calibrated OLG models, but neglect the positive effects of policy for generations alive today. Climate policy changes the future trajectory of consumption, and eventually leads to higher utility flows than the levels under Business as Usual (BAU). In models that emphasize conflict among agents living at different point in time, comparison of the consumption trajectories under BAU and under a climate policy depends on the social discount rate, a parameter (or function) about which there is considerable disagreement (Stern, 2007; Nordhaus, 2007). Our model emphasizes the potential conflict among agents living at the same point in time, and the substantial alignment of interests among agents living at different points in time, making the social discount rate is much less important.

There already exist two challenges to the conventional view that environmental policy requires sacrifices by those alive today. First, with multiple market failures, there may be "win-win" opportunities, so that correcting the externalities jointly makes it possible to protect the environment without reducing current consumption. Second, Foley (2009) notes that there may be opportunities to rebalance society's investment portfolio, reducing saving of man-made capital, e.g. industrial infrastructure, and increasing saving of environmental capital in such a way that leaves all generations better off than under BAU. In this situation, each generation is better off under the environmental policy. Rezai et al. (2011), using a model that resembles DICE (Nordhaus, 2008), find that Foley's conjecture is plausible. Proponents of the conventional view recognize this possibility.[1]

3 Model

Our model contains a single distortion and a single endogenously changing stock, the environment, and therefore excludes the possibility of win-win situations and of a reallocation of the savings portfolio. Agents alive in the current period have only one way to influence the future, by changing their current use of the environmental stock. Agents care only about their own lifetime welfare. Thus, questions about the social discount rate are essen-

[1]For example, Nordhaus (2007) discusses an option that "... keeps consumption the same for the present but rearranges societal investments away from conventional capital (structure, equipment, education and the like) to investments in abatement of greenhouse gas emissions (in 'climate capital', so to speak)."

tially irrelevant, although each generation's discounting of its own future consumption still matters. The assumptions of a single endogenous stock and non-altruistic agents make the model tractable, and also bias the model against finding that environmental policy improves welfare for each generation. However, we find that an environmental tax, with appropriate allocation of tax revenues, creates a Pareto improvement and can be implemented and sustained in a political economy equilibrium.

We use a two-sector Ricardo-Viner discrete time overlapping generations model. In each period t a cohort of constant size 1 is born. We suppress time subscripts when convenient. Agents live two periods; they are risk-neutral, maximize their intertemporal additive, homothetic utility, and have no bequest motive.

One sector, "manufacturing", produces a good M using labor and a sector-specific input, capital. The stock of capital is fixed, $K \equiv 1$; later we relax this assumption. The other sector produces a good F using labor and an endogenously changing resource stock, x. There are perfect property rights for the stock of manufacturing capital, and no property rights for the resource stock. Labor is perfectly mobile, and in the absence of an environmental policy competes away all rent in the resource-intensive sector.

Young agents receive a wage, income from the resource sector, and possibly a share of tax revenues. They divide their income between consumption of the two goods and purchase of manufacturing capital. The old generation earns the profits of its manufacturing firm, the proceeds from selling the firm, and its share of the tax revenue. Because agents are non-altruistic, the old generation consumes all of its income.

The labor and commodity markets are competitive and clear in each period. Employment in the resource sector equals L and free movement of labor between the sectors ensures that the return to labor there equals the manufacturing wage. Manufacturing is the numeraire good and the relative price of the resource-intensive good is P. Output in the resource-intensive sector is $F = Lf(x)$ with x the stock of the resource and $f(x)$ the output per unit of labor. The function $f(x)$ is increasing and weakly concave. Manufacturing output is $M = m(1 - L)$ where m is increasing and strictly concave, so that there are profits (rent) in this sector. Manufacturing firms are the only asset of the economy. They are owned by the old generation and sold to the young generation.

The open access of the resource sector means that too much labor moves to this sector. This misallocation can be reduced by imposing an ad-valorem tax, T, on production of the resource-intensive good. The revenue accruing to workers in the resource sector, under the tax, equals $P(1-T)Lf(x)$. Society returns the tax revenue, $R = PTLf(x)$, in a lump sum, but possibly different shares, to the young and old generations.

We examine the distributional effect of such a tax. The tax reduces the returns to labor in the resource sector and, hence, reduces the misallocation of labor. This reallocation of labor increases the resource stock in future periods. As labor flows into the manufacturing firm, wages fall and manufacturing output and nominal profits rise. A no-arbitrage condition introduced below implies that the value of the firm is equal to the discounted sum of future profits, deflated by a price index. Under mild restrictions, the environmental tax increases these future profits, thereby increasing the asset price and benefiting the old. However, the tax also increases the relative price P, reducing the current real rental rate below its original level. The current real wage also necessarily falls, due to the decrease in the nominal wage and the increase in the price level. By increasing the future resource stock, the tax also affects future generations.

The endogenous variables σ_t and $\tilde{\sigma}_{t+1}$ are the present and the expected next-period value of the firm; χ_t and $\tilde{\chi}_{t+1}$ are the present and expected next-period share of the tax revenue received by the young; and $\tilde{\pi}_{t+1}$ is the expected next-period manufacturing profit. We assume intertemporal additive utility, with the single period utility function $u\left(c_{F,t}, c_{M,t}\right)$, where $c_{i,j}$ is the consumption level of good i at time j. The agent's pure rate of time preference is ρ. The lifetime decision problem of the representative agent who is young in period t, is

$$\max_{c_{F,t}, c_{M,t}, c_{F,t+1}, c_{M,t+1}} u(c_{F,t}, c_{M,t}) + \frac{1}{1+\rho} u(c_{F,t+1}, c_{M,t+1})$$

subject to the budget constraint in the first and second period of their life:

$$w_t + \chi_t R_t \geq P_t \; c_{F,t} + c_{M,t} + \sigma_t, \text{ and}$$

$$\tilde{\sigma}_{t+1} + (1 - \tilde{\chi}_{t+1})\tilde{R}_{t+1} + \tilde{\pi}_{t+1} \geq \tilde{P}_{t+1} c_{F,t+1} + c_{M,t+1}.$$

Agents take as given, or have rational point expectations of:

$$w_t,\, P_t,\, \tilde{P}_{t+1},\, \sigma_t,\, \tilde{\sigma}_{t+1},\, \chi_t,\, \tilde{\chi}_{t+1},\, R_t,\, \tilde{R}_{t+1},\, \rho,\, \tilde{\pi}_{t+1}.$$

The young agent dedicates all of her time to working and the old agent manages the manufacturing firm.

To solve for the static equilibrium, we choose Cobb-Douglas functional forms for the utility function, u, and the production function in manufacturing, m. Period t utility and manufacturing output are:

$$u(.) = c_{F,t}^{\alpha} c_{M,t}^{1-\alpha}; \qquad M = m(L) = (1-L)^{\beta},$$

with α the constant budget share for the resource-intensive good and $\beta < 1$ labor's share of the value of manufacturing output. Indirect utility is linear in the expenditure level e

$$v(e,P) = \left(\frac{\alpha e}{P}\right)^{\alpha} \left(\frac{(1-\alpha)\,e}{1}\right)^{1-\alpha} = \alpha^{\alpha}\,(1-\alpha)^{1-\alpha}\,P^{-\alpha} e = \mu P^{-\alpha} e,$$

with $\mu \equiv \alpha^{\alpha}\,(1-\alpha)^{1-\alpha}$.

The assumption of identical homothetic preferences implies that the share of income devoted to each good is independent of both the level and distribution of income; prices do not depend on the distribution of income. The ratio of demand for both goods is a function of this price. The requirements that workers are indifferent between working in either sector, $P(1-T)f(x) = w$, and that manufacturing firms maximize profits, determine the wage, the allocation of labor, and supply of both goods. The relative price, P, causes product markets to clear. These equilibrium conditions for the labor and product markets lead to the following expressions for the values of w, L, and P:

$$\begin{aligned} L &= \tfrac{1-T}{\frac{1-\alpha}{\alpha}\beta + 1 - T}, \qquad w = \beta\left(1 + \tfrac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta} \\ P &= \tfrac{w}{(1-T)f(x)} = \frac{\beta\left(1+\frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}}{(1-T)f(x)} \equiv p(x,T). \end{aligned} \tag{1}$$

Under Cobb-Douglas technology and preferences, the equilibrium allocation of labor and the wage do not depend on the resource stock, x, only on the tax T and the parameters α and β. However, the equilibrium commodity price depends on x via the function $f(x)$. Firms' profits, π, the tax revenue, R, and the sectoral values of output, PF and M, depend also only on T and model parameters:

$$\begin{aligned} \pi = \tfrac{1-\beta}{\beta} w(1-L), &\qquad R = \tfrac{T}{1-T} L w, \\ M = (1-L)^{\beta}, &\qquad PF = \tfrac{\alpha}{1-\alpha}(1-L)^{\beta}. \end{aligned} \tag{2}$$

Systems (1) and (2) determine the static equilibrium of the economy.

3.1 Relation to Integrated Assessment Models

The Introduction notes that the tax reduces aggregate current utility, as in one-commodity Integrated Assessment Models (IAMs). In addition, the model treats capital an labor symmetrically in that stricter environmental policy reduces both the real wage, $\mu P^{-\alpha} w$, and the real rental rate, $\mu P^{-\alpha} \pi$, whereas a higher environmental stock increases current utility and the real wage and rental rate. The following proposition summarizes these claims; the appendix contains proofs not found in the text.

Proposition 1 *(i) An increase in the tax at time t reduces aggregate period-t utility. (ii) For a predetermined level of the environmental stock, a higher tax decreases both the real wage and the real rental rate. (iii) A higher environmental stock increases utility and both the real wage and the real rental rate.*

The tax- and stock-induced changes in the real rental rate are general equilibrium effects. The higher tax increases, and the higher stock leaves unaltered, the *nominal* rental rate. However, the change in the real rental rate also depends on the change in the price index P; this change more than offsets the wage effect. Most IAMs posit a somewhat *ad hoc* relation between environmental stocks and the level of real national or world income. The general equilibrium framework shows how changes in the environmental stock affects the real price of factors that do not directly depend on this stock – the manufacturing capital in our model.

Figure 1 uses a production possibility frontier to illustrate the welfare effect of environmental policy both in standard IAMs and in our OLG model. Under BAU, current consumption is at point A, a level that maximizes current aggregate utility, ignoring the environmental externality. The tax moves consumption to point B, where current aggregate utility is lower. Therefore, at least one of the two agents has lower current utility at B than at A.

Figure 1 illustrates the conventional view that environmental policy creates a conflict between those alive today and those alive in the future. The consumption path under BAU moves along the curve from A to A', a trajectory that incorporates changes in both environmental and man-made capital stocks, including technological change (introduced in Section 7). The consumption trajectory under the environmental policy moves along the curve from point B to B''. Agents alive at the initial time have higher utility under trajectory AA', and those alive later have higher utility under trajectory

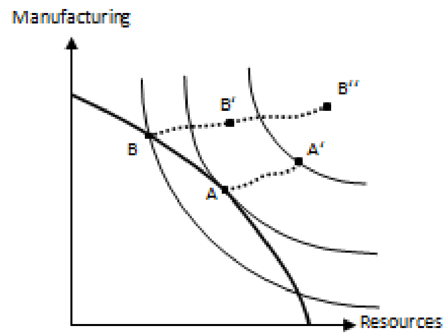

Figure 1: Consumption expansion paths under BAU $(A - A')$ and under an environmental policy $(B - B' - B'')$

BB'', so in conventional models a welfare comparison depends on the social discount rate.

The two previous challenges to the conventional view, the existence of win-win situations or the possibility of reallocating the investment portfolio, imply that environmental policy moves society from trajectory AA' to trajectory $B'B''$. With this move, agents in every period have higher utility under environmental policy. Our model rules out both of the previous challenges: there are no win-win opportunities, and the assumption that the environment is the only endogenously changing stock excludes the possibility of reallocating investment across stocks.

Environmental policy in our model does lower aggregate utility of consumption in the first period. The current old live for a single period, so the policy increases their lifetime welfare if and only if it increases their utility in the current period. The current young will also be alive in the next period. Even if the policy lowers their current utility, their lifetime welfare can increase if their utility in the next period increases sufficiently.

Although a higher tax and a higher environmental stock have the same qualitative effects on the real wage and the real rental rate there is a fundamental asymmetry between the two factors: the *price* of capital depends on future rental rates, whereas the price of labor depends on only its current value of marginal productivity. Current owners of capital benefit from the future increases in productivity created by the environmental policy, even though they are not alive to enjoy them directly. Absent transfers, current owners of labor benefit from these future productivity increases only to the extent that they are alive to enjoy them.

3.2 The asset price

We need to specify the dynamics of the natural resource and the relation between the environmental stock and labor productivity in order to determine the effect of environmental policy on the asset price. We assume that the average product of labor in the resource sector is linear in the stock ($f(x) = \gamma x$ for a constant $\gamma > 0$) and that the resource stock obeys a logistic growth function. The resource transition equation is

$$\begin{aligned} x_{t+1} &= x_t + r x_t \left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma x_t = \left(1 + r\left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma\right) x_t \\ &= \left(1 + \bar{r}_t(T_t, x_t)\right) x_t; \text{ with } \bar{r}_t \equiv \left(r\left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma\right), \end{aligned} \tag{3}$$

with r the intrinsic growth rate, C the carrying capacity of the resource, and $\bar{r}$ the endogenous growth rate of the resource. A higher tax conserves the resource because $\frac{dL_t}{dT_t} < 0 \Rightarrow \frac{d\bar{r}_t}{dT_t} > 0 \Rightarrow \frac{dx_{t+1}}{dT_t} > 0$.

We restrict parameter values to ensure that under BAU there exists an interior steady state, x_∞, to which trajectories beginning near that steady state converge monotonically. The necessary and sufficient conditions for this are $1 > \frac{d(1+\bar{r})x}{dx} > 0$, evaluated at $T = 0$, $x = x_\infty$. These inequalities are equivalent to

$$1 < \varsigma < 2 \text{ with } \varsigma \equiv r + \frac{\beta(1-\alpha) + \alpha(1-\gamma)}{\beta(1-\alpha) + \alpha}. \tag{4}$$

The unique non-trivial steady state stock of the resource is

$$x_\infty = C\left(1 - \frac{\gamma L(T_\infty)}{r}\right) = C\frac{\varsigma - 1}{r}. \tag{5}$$

The BAU trajectory is monotonic if and only if the initial condition satisfies $x_0 \leq \frac{1}{\varsigma - 1} x_\infty$.

The young buy manufacturing firms from the old; the asset price affects welfare through expenditure. Systems (1) and (2) enable us to state the young and old generation's expenditure levels, e^y and e^o, as functions of current tax T and the asset price, $\sigma(x, \mathbf{T})$, where $\mathbf{T}$ is the tax trajectory:

$$e^y = w(T) + \chi R(T) - \sigma(x, \mathbf{T}) \text{ and } e^o = \pi(T) + (1-\chi)R(T) + \sigma(x, \mathbf{T}). \tag{6}$$

A no-arbitrage condition requires that the young's marginal loss in utility from purchasing a unit of the asset in the current period equals their marginal gain in utility from having that asset in the next period. This condition determines the demand for the asset as a function of its current price and expectation of next period rental rate and price. This demand function, and the fixed (or exogenously changing) supply of capital, determine the current asset price as a function of expected next period rental rate and price, leading to:

Proposition 2 *The price of a unit of capital is equal to the infinite sum of future discounted utility arising from the firm's future profits, evaluated at current prices.*

A policy change that, for example, increases the asset price, benefits the current asset owners, the old. The changed asset price has no effect on the welfare of asset purchasers, the young. The no-arbitrage condition described above implies that the young pay exactly what the asset is worth to them. Although the change in asset price changes their current expenditures, the offsetting change in future receipts leads to a zero change in their welfare:

Corollary 1 *(i) An unanticipated change in the asset price does not affect the lifetime utility of current and future young generations. (ii) Unanticipated changes in the asset price affect only the current old generation.*

4 Welfare Effects of a Tax

Under BAU, the environmental tax is identically 0. Consider an arbitrary non-negative tax trajectory, the vector $\bar{\mathbf{T}}$, with element $\bar{T}_i \geq 0$. Strict inequality holds for at least one i, including $i = 0$. The index i denotes the number of periods in the future, so $i = 0$ denotes the current period. A non-negative perturbation of the zero tax BAU policy is $\mathbf{T} = \varepsilon\bar{\mathbf{T}}$, with $\varepsilon \geq 0$ the perturbation parameter. A larger ε therefore is equivalent to a higher tax policy. We adopt the assumption that $\bar{T}_0 > 0$, i.e. that the environmental policy begins in the current period, because consideration of delayed policies yields only obvious results. In this section we set the fraction of tax revenue given to the young, χ, to be a constant, an assumption we revisit in Section 6. The following proposition provides a sufficient condition for a non-negative perturbation of the BAU policy to improve the welfare of the old generation.

Proposition 3 *For all* $\chi \in [0,1]$*, a sufficient condition for the old generation to benefit from a small tax increase is that the initial value of the environmental stock (when the policy begins) satisfies* $x_0 \leq \frac{1}{\varsigma - 1} x_\infty$*, where* ς *and* x_∞ *are defined in equations (4) and (5). The old generation's welfare increases in its tax share,* $(1-\chi)$*.*

The sufficient condition, stated in terms of the initial value of the environmental stock, ensures that the BAU stock trajectory approaches the BAU steady state monotonically. Inspection of the proof shows that the old can benefit from a tax even when this restriction does not hold. We have the following immediate result.

Corollary 2 *Under the condition stated in Proposition 3, the tax leads to a fall in first-period welfare of the present young generation.*

Proof. Propositions 1 and 3 state that aggregate current welfare falls while welfare of the old generation rises. Therefore, first-period welfare of the current young must fall. ■

In general, a price change creates winners and losers. The OLG framework shows that a policy that discourages over-use of a resource benefits asset holders and in the first period harms the young agents. Of course, the policy also changes the consumption of the current-young in the next period, thereby creating the possibility of higher *lifetime* welfare. To avoid uninteresting complications, we assume for the rest of this section that $\bar{T}_1 = \bar{T}_0 > 0$.

Proposition 4 *(i) For a constant* $\chi \in [0,1]$*, a small increase in tax rates (larger* ε*) increases lifetime welfare of the present young generation if and only if: (a) it receives less than the entire tax revenue while young* ($\chi < 1$)*, and (b)* $\bar{r}(0, x_t) > (1+\rho)^{\frac{1}{\alpha}} - 1$*, i.e. the pure rate of time preference is less than the positive welfare effect of lower prices due to the higher resource stock. (ii) If the renewable resource is falling on the BAU trajectory and* $\chi < 1$*, the tax policy lowers the young generation's lifetime welfare.*

The case relevant for most problems involving environmental stocks, and climate policy in particular, is where the resource is being degraded, i.e. where $\bar{r}(0, x_t) < 0$. In this circumstance, condition (b) in the Proposition fails, and the tax policy necessarily reduces the lifetime welfare of the young as Part (ii) states.

Even if a small tax potentially harms the young, it makes sense to ask whether the young would prefer to receive a larger share of tax revenue when young or old, holding the tax fixed.

Proposition 5 *The young generation prefers a constant $\chi = 0$ (i.e. receipt of all tax revenue when it is old) if and only if it benefits from a tax introduction. If the policy lowers their welfare, they prefer to receive all of the tax revenue while young.*

Proposition 4 condition (b) states that the young benefit only if they can increase their welfare by shifting their income into the future. In this case, the young generation wants to shift all its tax receipts into the future, $\chi = 0$. If condition (b) does not hold and $\chi = 1$, the tax creates a zero first order effect on the young generation's welfare; the first order effects of a tax on the real wage and the tax revenue sum to zero.

Propositions 4 and 5 are based on the assumption that the old in each period receive the same share of tax revenue, i.e. that $1 - \chi$ is constant. That assumption is useful for understanding the distributional effect of environmental policy, but it is not reasonable as a policy prescription. The old in the period when the tax is imposed – unlike the old in any other period – capture the future benefits that are capitalized in the asset price (Corollary 1). In addition, the young in future periods benefit from a higher resource stock (relative to BAU) in both periods of their life; the young in the current period benefit from environmental protection in only the second period of their life. Therefore, it is reasonable to treat the old and the young in the period when the policy is introduced differently than their counterparts in future periods. In particular, the current young should receive a larger share of tax revenues, compared to the young in future periods. This favorable treatment makes it possible for the policy to improve the young generation's welfare, an observation that motivates the analysis in Section 6.

In summary, if the environmental problem is that the resource is below its 0-tax steady state and therefore recovering, but just not recovering sufficiently quickly, then the young potentially would support a tax that speeds recovery. In that circumstance, both the young and the old generations want all of the tax revenue to go to the old, under the constraint that the share is constant. In the more relevant circumstance where the environmental objective is to keep the resource from degrading excessively, the young would oppose a tax that helps to solve the problem. If such a tax were forced upon them, and the tax share χ were constant, they would prefer to receive all of the tax revenue while young. Thus, in the case that is relevant to most problems involving environmental stocks, this OLG model shows that there is a conflict between generations alive at the time society imposes the tax. The old generation favors the environmental policy because some of the future benefits of that policy are capitalized into the asset value. The current young obtain none of those capitalized benefits, and they do not live long enough to reap significant benefits from the improved environment.

5 Robustness

Having presented the model, we are now in a position to discuss its robustness. The discussion of Proposition 1 notes the model's relation to IAMs and its symmetric treatment of capital and labor: stricter environmental policy reduces real current income and reduces both the real wage and the real rental rate. The critical difference between capital and labor is that the price of capital reflects tax-induced *future* increases in real rental rates, whereas the price of labor, the wage, reflects none of the future increases in labor productivity. Asset owners therefore capture some of the future productivity gains due to the tax, whereas agents who sell their labor do not. The capital gains are the means by which markets transfer some benefits of environmental policy from the future to the present. This transfer is the basis for our result that such policy is likely to involve a conflict between generations alive when the policy is first imposed.

As with all IAMs (that we know of), there is a single type of capital, an assumption motivated by tractability not realism. We do not, of course, think that environmental policy increases the value of all assets; counterexamples are easy to find. These models simply assume that a better environmental stock increases the (real) return to an *aggregate* measure of capital, and that environmental policy improves the future environmental stock. It seems hard to disagree with these two assumptions.

The crucial assumption is that the stock of capital is fixed, or at least exogenous. The fixed stock means that the price of the asset responds to changes, relative to BAU, in the future environmental stock. If investment costs were nearly linear, the asset price would remain close to the (fixed) commodity price, and thus be insensitive to the future environmental stock. If capital depreciates quickly, then the price of capital, at the beginning of the period during which the environmental policy begins, is approximately equal to profits in the current period, and again insensitive to future environmental stocks. Thus, the model is most applicable if adjustment costs are significant and depreciation of capital low, relative to changes in environmental stocks. These conditions hold almost exactly for some assets, such as land, and not for others, such as manufacturing plant. The empirical relevance of the model therefore depends to a great extent on the degree of convexity of adjustment costs and the relative speeds of change in capital and environmental stocks. The model here helps to identify previously ignored channels that relate environmental policy to welfare; it provides a prelude to analysis of a more general model, the subject of current research.

6 Transfers

Here we consider the role of transfers when under BAU the resource is degrading, $\bar{r}(0, x_t) < 0$. The proof of Proposition 4 shows that a small tax has only a second order welfare effect on the young if they receive all of the tax revenue while young ($\chi = 1$). We noted above that the old obtain a first order welfare gain even if they receive none of the tax revenue. Given these two results it is not surprising that for a small tax, it is always possible for the old generation to make a transfer to the young, in addition to giving them all of the tax revenue, so that both generations are better off. This means of compensating the young requires that the old give them a portion of the tax-induced increase in the asset value. It might be politically difficult to achieve such a transfer.

An alternative means of compensating the young is to give them a higher share of tax revenue, compared to the future young. One way to do this is to hold the tax share parameter $\chi < 1$ constant, but give the first-period young the fraction ξ of the old generation's share of tax revenue. This transfer scheme allows the first period old to keep all of the capital gains and the fraction $(1 - \chi)(1 - \xi)$ of tax revenue. In this way, the future young (rather than the current old) compensate the current young to make the latter willing to accept the tax policy. We state this formally:

Proposition 6 *For constant $\chi < 1$ there exists a tax transfer rate $0 \leq \xi^{crit} < 1$ from the present old to the present young such that with $\xi > \xi^{crit}$, a small tax policy with $\bar{T}_0 = \bar{T}_1 > 0$ creates a Pareto improvement.*

Because the young gain under this tax and transfer, an argument parallel to that which establishes Proposition 5 implies that for any $\xi > \xi^{crit}$, both generations prefer $\chi = 0$. In the bargaining model studied in Section 8 we therefore emphasize the case $\chi = 0$.

The fact that a tax and transfer combination creates a Pareto improvement for the generations alive at the time society imposes the policy is noteworthy because it arises in a model that appears biased in favor of finding that an environmental policy harms some generation. Agents alive at the time the policy is imposed do not care about the welfare of future generations. In addition, they have only one means of accumulation: protecting the environment. That protection always requires that aggregate first period utility of consumption falls.

Generations sufficiently far in the future are also better off due to a small tax. A small tax has only a second order effect on "static efficiency", the efficiency calculation that holds the trajectory of the resource stock fixed. However, the tax has a first order effect on the steady state resource stock, and that increased stock creates a first order welfare gain in the steady state. Absent transfers, the tax is more likely to benefit future generations compared to the current young generation: the tax-induced higher stock benefits each of the future generations in two periods, whereas it benefits the current young generation in only one period. (See Appendix B.1 for details.)

7 Exogenous Productivity Growth

In the context of most environmental problems, the natural resource is degrading on the 0-tax trajectory. In our model of constant productivity and capital, the world becomes poorer and future generations have lower welfare on that trajectory. This section introduces exogenous productivity growth in both sectors. Let $a \geq 0$ be the growth rate of total factor productivity in manufacturing and $b \geq 0$ the growth rate of efficiency in output per unit flow of the resource. Sectoral output is

$$M_t = e^{at}(1 - L_t)^{\beta} \quad \text{and} \quad F_t = e^{bt} L_t \gamma x_t.$$

The inequality $a > 0$ can also be interpreted as exogenous growth in the stock of capital. The extraction of the resource is still $L_t \gamma x_t$ (not $e^{bt} L_t \gamma x_t$). This model of resource productivity growth implies that each extracted unit of the resource increases the supply of the resource-intensive commodity. If we think of the resource as being energy, the assumption means that the economy becomes less energy intensive. At a constant stock level, this form of productivity growth implies growth rates for utility of $e^{(1-\alpha)a+\alpha b}$, for the price level of $e^{(a-b)}\left(1+\bar{r}(T_t, x_t)\right)$ and for all other variables (w_t, R_t, and π_t) of e^a. For the following proposition we assume that $\chi \in (0,1)$ is constant and that there is no transfer between generations, i.e. $\xi = 0$.

Proposition 7 *A larger value of $a-b$ increases the stringency of the necessary and sufficient condition under which a small constant tax increases the welfare of the young.*

Under proportional growth ($a = b$), the condition for the young to benefit from the tax is the same as when $a = b = 0$. The welfare effect of the tax, for the young, depends on the change in the price level. A *ceteris paribus* increase in $a-b$ increases the next period relative supply of the manufacturing good, thereby increasing the future relative price of the resource-intensive good, P_{t+1}. The higher price lowers the marginal utility of next period income, making it "less likely" that the young are willing to forgo income today in order to have higher income in the next period. For $a > b$, the young would require a higher transfer from the old in order to agree to the tax. If, however, the productivity in the resource sector grows much faster than in the manufacturing sector ($b >> a$), the young might support a tax even when the resource is shrinking on the 0-tax trajectory, and in the absence of a transfer.

8 Political Economy Equilibria

Both current generations can gain from a tax, given proper allocation of tax revenues. To find the equilibrium tax and transfer levels and to explore the political economy details, we calibrate the model and solve it numerically for a specific political economy setting. In this scenario, we assume that in each period the old and the young solve a cooperative game, i.e. they choose a tax to maximize their joint surplus. We refer to this scenario as "efficient bargaining". We compare the efficient bargaining outcome to the outcome under a social planner that chooses the tax sequence to maximize the present discounted value of the infinite sequence of future single-period aggregate utility, using the discount rate ρ.

We also experimented with another scenario to test the model's sensitivity to the political economy structure, and found that the results are similar. In particular, we considered a setting that restricts the bargaining opportunities in the game between the young and the old generation in each period; we report some features of that scenario in subsequent footnotes. We refer to that scenario as "inefficient bargaining".[2] Beliefs about the future bargaining outcomes constrain the current bargaining game. Because of this constraint, there is no reason to suppose that the outcome under efficient bargaining is "better" than under inefficient bargaining. Indeed, by some measures the outcome is worse under efficient bargaining.

These political economy models show that there remains some conflict across generations that live during different periods. Generations in the future always prefer that previous generations use a larger tax, to generate a larger environmental stock. Our claim is simply that starting with a major unsolved environmental problem, here represented by a zero BAU tax, all generations can be made better off when agents alive at each point are able to bargain amongst themselves, even when they do not care about the welfare of those who will live in the future.

8.1 Calibration

The parameter α is the share of the resource-intensive commodity in the consumption basket. We set $\alpha = 0.2$, equal to the approximate share of non-durable good consumption in the US (see US-NIPA, 2010). The wage share in manufacturing, β, in the US is around 0.6. We set the annual pure rate of time preference at $2\%/year$ which gives $\rho = 1$ assuming one period lasts 35 years.

In light of the climate policy motivation, we model the renewable resource as easily exhaustible and slowly regenerating; these choices reflect the view that climate change is a serious environmental problem. We choose units of

[2]For the model of inefficient bargaining, we set $\chi = 0$ for the reasons discussed in Section 6, and allowed the old in the first period to propose a transfer rate ξ. Conditional on this choice, the old and the young each propose a constant tax. Due to inertia, society chooses the smaller of these two taxes. We then confirmed numerically that this tax is time consistent. Future young generations would like to lower the tax and future old generations would like to increase it, but the welfare gain that either achieves is insufficient to compensate the other. Therefore, no proposed change achieves consensus. The belief in the initial period that the tax will be constant is therefore confirmed by the equilibrium. Details available on request.

the resource stock, x, such that its carrying capacity is normalized to one, $C = 1$, so that x equals the capacity rate. The productivity parameter γ equals the inverse of the amount of labor that would exhaust the resource in a single period, starting from the carrying capacity $x_0 = 1$. We set $\gamma = 3.33$ and $r = 1.37$ which is equivalent to an uncongested growth rate of $2.5\%/year$. On a 0-tax trajectory the resource continues to degrade to a steady state of $x_\infty = 0.285$. Equation system (7) summarizes the parameter values:

$$\alpha = 0.2;\ \beta = 0.6;\ \rho = 1;\ r = 1.37;\ \gamma = 3.33. \tag{7}$$

For this parameter set, the old generation has a higher expenditure level than the young under BAU for any stock level. Here, the asset-rich and the asset-poor correspond to the rich and the poor. The BAU trajectory is monotonic if and only if $x_0 \leq 0.73$. For larger initial conditions, the BAU trajectory drops below the steady state in the first period and then approaches the steady state monotonically from below.

8.2 Efficient Bargaining

Here we assume that in each period the current young and the current old bargain over the tax in order to maximize their joint lifetime welfare; in this sense, bargaining is efficient. Agents recognize that future generations have the same flexibility; in particular, the taxes in future periods are conditioned on the future value of the directly-payoff-relevant state variable, the environmental stock. Agents currently alive are able to influence future policies by influencing the state variable that they bequeath to the future, but current agents cannot choose future policies. That is, we consider a stationary Markov Perfect equilibrium (MPE) in the dynamic game amongst the succession of generations. The transfer from the future to the present occurs via the asset price; future taxes affect this price. The cost, to those currently alive, of the efficiency in bargaining, is a possible loss of commitment ability, relative to bargaining environments where friction makes it harder to change policies. Consequently, all agents may be worse off under efficient bargaining, compared to the particular inefficient bargaining structure that we alluded to above.

If the tax revenue is non-zero and χ is unbounded, any transfer between generations alive at a point in time can be achieved by a suitable choice of the current value of χ; here, χ is perfect substitute for lump sum transfers.

In general, the equilibrium values of both χ and T are functions of the environmental stock. In order to see that these two policy functions are mutually dependent, consider two worlds with constant exogenous χ and no lump sum transfers; $\chi = 1$ in one world and $\chi = 0$ in the other. Even if agents believed that the future taxes would be determined by the same policy function, they would have different incentives in choosing the current tax rate, simply because the current young would obtain a different fraction of the next-period tax revenue in the two worlds; that tax revenue depends on the next-period stock, which depends on the current tax. Therefore, the equilibrium tax functions must be different in the two worlds. The current value of χ affects only the distribution of income, not the equilibrium tax. However the anticipated next-period value of χ does affect the current equilibrium tax.

To solve a model in which both taxes and the division of tax revenue are endogenous, we would need to specify the particulars of the game, e.g. the factors that determine the division of surplus. As an alternative, we solve the tax-setting game conditional on a fixed value of χ, and then check for the sensitivity of the equilibrium with respect to χ. Even with constant χ, we find that the equilibrium tax policies are non-monotonic in the environmental stock, a fact that makes it difficult to determine *ex ante* the relation between χ and the incentive to tax in the current period. Numerical results show that the equilibrium is insensitive to the two obvious choices, $\chi = 1$ and $\chi = 0$. We therefore present here the derivation and results for $\chi = 0$; this is the obvious choice, for reasons explained in Section 6. Appendix B.2 contains sensitivity results.

The nominal value of national income in period t is $Y(T_t) = P_t F_t + m(1 - L_t)$ and the aggregate utility in period t (real national income) is $\mu p^{-\alpha}(x_t, T_t) Y(T_t)$; see the proof of Proposition 1 for the explicit form of $Y(T_t)$.

Taking as given $\chi = 0$, our goal is to find the equilibrium stationary tax function, denoted $T_t = \Upsilon(x_t)$. The Nash condition requires that given agents' belief that $T_{t+i} = \Upsilon(x_{t+i})$ for $i > 0$, the equilibrium decision for the agents choosing the current tax is $T_t = \Upsilon(x_t)$. Ownership of the asset entitles the owner to profits and revenue from the sale of the asset after production. By purchasing the asset from the old in period t, the agent who is young in period t obtains the utility derived from profits and asset sales when she is old. We denote the level of utility obtained from the sale of assets in the next period as $\bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1}))$, and define this function recursively, using

$$\bar{\sigma}(x_t, T_t) = \frac{1}{1+\rho} \left\{ p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(\Upsilon(x_{t+1})) + \bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1})) \right\}. \tag{8}$$

Equation (8) states that the utility that the old generation receives in period t, from the sale of assets to the young generation in that period, equals the young generation's present value of the utility from next-period profits, plus the utility from their future sale of the asset. This equation is the utility analog of the no-arbitrage condition used in the proof of Proposition 2 (equation (13)) obtained by defining $\bar{\sigma}_t = P_t^{-\alpha}\sigma_t$.

Because $\chi = 0$, the t-period young also obtain all of the tax revenue in the next period; using the second equation in system (2), we write this revenue as $R(\Upsilon(x_{t+1}))$; the present value of the utility of this revenue is $\frac{1}{1+\rho}\mu p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))R(\Upsilon(x_{t+1}))$

The bargaining equilibrium in period t is the solution to the optimization problem

$$\max_{T_t} U^o + U^y =$$

$$\max_{T_t} \left\{ \mu p^{-\alpha}(x_t, T_t) Y(T_t) + \mu\bar{\sigma}(x_t, T_t) + \tfrac{1}{1+\rho}\mu p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))R(\Upsilon(x_{t+1})) \right\} \tag{9}$$

subject to

$$x_{t+1} = (1 + \bar{r}_t(x_t, T_t))\, x_t. \tag{10}$$

Equation (9) states that the objective is to maximize the lifetime utility of the current old and the current young generation. This maximand equals the utility value of current national income, plus the present value of the utility value of owning the asset in the next period and receiving the tax revenue.

The primitives of the model lead to explicit expressions for the functions $p(x, T)$ and $Y(T)$. Equation (8) recursively determines the function $\bar{\sigma}(x_t, T_t)$. Agents at time t take the functions $\Upsilon(x_{t+1})$ and $\bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1}))$ as given, but they are endogenous to the problem. We obtain a numerical solution using the collocation method and Chebyshev polynomials (Judd, 1998; Miranda and Fackler, 2002); see Appendix B.2.

Figures 2 and 3 summarize the numerical solution to the problem in equations (9) and (10) for the parameter values of equation system (7). The figures also contain information about a social planner's problem, discussed

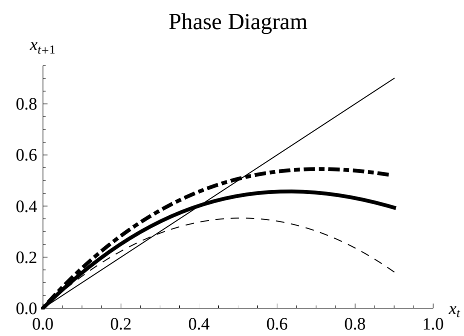

Figure 2: The phase diagram for the resource stock under the efficient-bargaining tax policy (solid), the 0-tax scenario (dashed), and the social planner (dot-dash).

in the next subsection. Inspection of the discrete time phase diagram in Figure 2 shows that for any current stock, the next period stock is higher under the efficient-bargaining tax compared to under BAU: the environmental policy protects the resource stock. The steady state stock level under efficient bargaining is 0.38, compared to the BAU level of 0.28. Under the efficient-bargaining tax, the stock trajectory is a monotonic function of time. In contrast, under BAU, for large initial values of x, the subsequent level of x is below the steady state. In this situation, the BAU trajectory first overshoots the steady state and then approaches the steady state from below.[3]

The possibility of overshooting helps to explain why the equilibrium tax policy is (slightly) non-monotonic in the stock (Figure 3), and also why the asset value, in units of utility, is monotonic in the stock under the tax policy, but non-monotonic under BAU. (To conserve space, this figure is not presented.) At high values of the resource stock, a high tax prevents the stock from overshooting the steady state, as would occur under BAU. At low values of the resource stock, a high tax helps the resource to regenerate. The equilibrium tax therefore reaches a minimum for an intermediate value of the stock. Under BAU, the possibility of overshooting causes the asset value to be low at high stock values; the asset value is also low when the low

[3]In the inefficient bargaining model described in the previous footnote, the steady state level is $x_{\infty}^{\text{ineff}} = 0.46$. The introduction of frictions in political decision-making increases the tax rate, benefiting future generations. This steady state level is close to the one chosen by the social planner, $x_{\infty}^{\text{social}} = 0.51$.

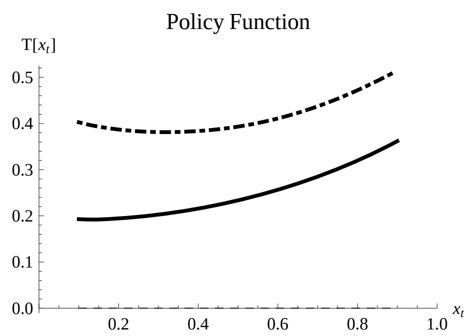

Figure 3: The policy function under efficient bargaining (solid) and under the social planner (dot-dash).

resource stock leads to low equilibrium utility. Under efficient bargaining, the equilibrium adjustment of the tax ensures that a higher resource stock leads to higher utility value of the asset.

Figure 4 shows agents' welfare gain under the efficient-bargaining tax, relative to BAU levels. For future generations ($i \geq 1$) we show the welfare gain of the young agent, and for the current generation ($i = 0$) we show the aggregate lifetime welfare gain for the current young and old generations. Section 4 explains why the generations alive when the tax is first imposed need to be treated differently than future generations. The dashed curve corresponds to the initial condition $x_0 = 0.45$ and the solid curve corresponds to $x_0 = 0.9$. For intermediate initial conditions, the welfare gain lies between these two curves. If the economy starts out slightly higher than the with-policy steady state, agents gain because under BAU welfare would fall to a low level as the resource degenerates. If the initial resource stock is far above the steady state, future generations additionally benefit because the tax prevents overshooting. The fact that overshooting is a problem for high but not for low initial stocks explains why the welfare gain falls when the initial stock is large. The aggregate gain to the first generations is $3 - 7\%$ and the steady state welfare gain is about 3%.

8.3 A social planner

We briefly consider the social planner's problem typically used in Integrated Assessment Models. The single period aggregate utility, as noted above,

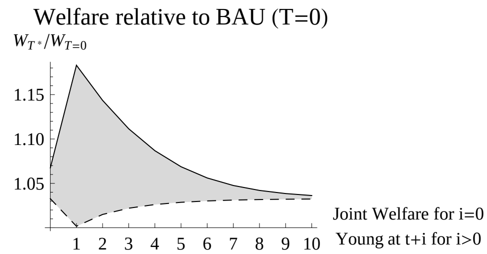

Figure 4: Welfare effects of efficient-bargaining tax policy relative to no-policy scenario for initial resource stock at steady state ($x_0 = 0.45$, dashed) and near full regeneration ($x_0 = 0.9$, solid).

is $\mu p(x_t, T_t)^{-\alpha} Y(T_t)$. Schneider, Traeger and Winkler (2010) explain the problems with using parameters that describe individual preferences in an OLG setting to calibrate a social discount rate. Here, for illustration, we take the social discount rate to be the individual agent's pure rate of time preference. The social planner's problem is

$$\max_{\{T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+\rho)^{-t} \, \mu p(x_t, T_t)^{-\alpha} Y(T_t)$$

$$\text{subject to } x_{t+1} = (1 + \bar{r}_t(x_t, T_t))\, x_t \quad \text{with } x_0 \text{ given.}$$

We obtain a numerical solution to the dynamic programming problem associated with this optimization problem, using the parameters above, the collocation method, and Chebyshev polynomials. The dot-dash graphs in Figures 2 and 3 show the phase portrait and the policy function for this social planner. The equilibrium stock and tax trajectories are higher under the social planner, compared to under efficient bargaining. This result is not surprising, given our assumption that agents have no bequest motive. The possibly surprising result is that the selfish agents' equilibrium tax is rather close to the social planner's tax. The social planner's steady state tax is $T = 0.40$, a level slightly higher than the tax that maximizes the steady state lifetime welfare of the young, $T = 0.38$.

In most IAMs, environmental policy involves a sacrifice by those currently alive. In our model, the social planner's policy lowers aggregate first period welfare and increases the welfare of those alive in future periods. Even here, however, *lifetime* aggregate welfare of the generations alive in the first period increases due to changes in asset prices.

9 Discussion

Many discussions about environmental policy start from the presumption that this policy requires current sacrifices in order to protect future generations. The two existing challenges to this presumption are that there may be win-win situations, and that it may be possible to reallocate current savings in order to make agents in each period better off. Both of these challenges require the social planner to have considerable power and acumen. We provide a different perspective, using a model that excludes both of the existing challenges to the conventional view.

In a general equilibrium OLG model, current owners of the *non-environmental* asset benefit from the increased asset value caused by the imposition of an environmental tax, even though they do not live long enough to enjoy the improved environment. Future generations benefit from the improved environment. The young generation that is alive when society imposes the policy loses, in the absence of transfers. This generation does not capture the increase in asset values, since it buys those assets. Moreover, it suffers current losses because the environmental tax decreases the real wage. The young generation is also not alive long enough to enjoy the benefit of the improved environment. Thus, there is a genuine intergenerational conflict, but it is not the conflict between those living today and those living in the distant future – the conflict that most of the literature emphasizes. Instead, it is the conflict between those who own assets and those who must purchase them: in our model, the old and the young currently alive.

The special status of the two generations currently alive makes it easier to design Pareto improving environmental policy. These two generations can strike a bargain between themselves without involving future generations, because the environmental improvements automatically leave future generations better off. The equilibrium policy in a political economy setting with efficient bargaining amongst those alive at a point in time is less conservationist than the policy under a particular alternative bargaining model that restricts the choice set. In this situation, greater within-period efficiency carries with it a costly decrease in the ability to make credible commitments, leading to a lower tax and greater emissions in equilibrium. We compared these political economy equilibria to the solution to the optimal control problem in which a social planner maximizes the present discounted stream of single period aggregate payoffs, using the intra-generational discount factor. This planning problem is analogous to the type of problem solved in IAMs, because it ignores the OLG structure. The tax policy in this optimal control problem is more conservationist than the equilibrium policy in either political economy setting. However, even under this more conservationist policy, the aggregate lifetime welfare of those alive in the first period is higher than under BAU, because the change in the asset price transfers some environmentally-induced benefits from the future to the present.

Although the academic literature on climate policy emphasizes conflict between current and distant future generations, the actual political dispute turns to a large degree on disagreements between developed and developing countries. Our model is too simple to accurately reflect the subtleties of that dispute, but the model does help to illuminate some important points. The developing nations are younger and poorer than developed nations. Our model shows that young and asset-poor agents have a just claim on compensation from old and asset-rich agents, if the former are to accept meaningful climate policy. It is not that the old rich can afford to and are morally obliged to make this compensation – a claim that may or may not be accepted. Rather, the old rich should make the compensation because the environmental policy benefits them and would, in the absence of the transfer, harm the young poor; in addition the old rich can finance the compensation using only a fraction of their increased benefits.

Both of the political economy models that we considered suggest that an environmental agreement emerges in equilibrium, a positive rather than a normative result. Why has a meaningful climate agreement thus far eluded us? There are two types of answer to this question. The first is that perhaps the world is better described by traditional IAMs, either because investment costs are nearly linear or because traded assets depreciate much more quickly than policy can alter environmental stocks. In these circumstances, the conflict between those alive today and those alive in the future is central; and those alive today are simply not willing to make the sacrifices or not able to overcome the obstacles to collective action, to a sufficient degree to benefit future generations. The second type of answer is that the debate about climate policy has been improperly framed, and that with better understanding of how this policy affects asset owners, there will be greater chance of an agreement. The view that climate policy will "cost us" seems natural; it may or may not be correct. This paper attempts to reframe the question of climate policy, in a way that focuses our attention on reasons to make transfers from the asset-rich to the asset-poor in order to induce the latter to agree to climate policy.

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A Proofs

A.1 Proof of Proposition 1

Proof. (Sketch) (i) Using systems (1) and (2), the nominal value of national income in period t is

$$Y\left(T_t\right) = P_t F_t + m\left(1 - L_t\right).$$

We multiply nominal national income by $\mu P^{-\alpha}$ to convert dollars to utils; $P = p(x_t, T_t)$ is a function of both the tax and the environmental stock. The single period aggregate utility is

$$\begin{aligned} U\left(x_t, T_t\right) &\equiv \mu p(x_t, T_t)^{-\alpha} Y(T_t) = x_t^{\alpha} \omega\left(T_t\right), \\ \text{with } \omega\left(T_t\right) &\equiv \mu \left(\frac{\beta \left(1 + \frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}}{(1-T)\gamma} \right)^{-\alpha}. \end{aligned}$$

Differentiating with respect to T and simplifying gives, for $T \neq 0$,

$$\frac{dU}{dT} = -\mu P^{-\alpha} \frac{(1-\alpha)\beta L T}{(1-T)^2} Y < 0. \tag{11}$$

(ii and iii) The tax decreases the nominal wage, w, and increases the equilibrium relative price, P, and therefore decreases the real wage. A higher stock does not affect the nominal wage but it decreases the equilibrium relative price, so it increases the real wage.

The real rental rate is $\mu P^{-\alpha}\pi$. The tax lowers the equilibrium nominal wage, increasing nominal profits, π, but it also increases the commodity price. Using the fact that preferences are homothetic and that the wage share is constant, we have

$$\mu P^{-\alpha}\pi = \mu P^{-\alpha}(1-\beta)\frac{Y}{1-\alpha}.$$

Differentiating this with respect to T gives, for $T \neq 0$,

$$\frac{d\mu P^{-\alpha}\pi}{dT} = \mu \frac{1-\beta}{1-\alpha} \frac{dP^{-\alpha}Y}{dT} = -\mu P^{-\alpha} \frac{(1-\beta)\beta L T}{(1-T)^2} Y < 0.$$

A higher stock does not alter nominal profits, but decreases the commodity price, thereby increasing real profits. ■

A.2 Proof of Proposition 2

Proof. The subscript on $\mathbf{T}_t$ denotes that the first element of the trajectory of taxes is the tax in period t. The price of a firm this period is σ_t and the expectation of the next-period price is $\tilde{\sigma}_{t+1}$. In equilibrium the young generation buys one firm today and sells it in the next period. With intertemporally additive, homothetic lifetime utility, the present value of total utility of the young agent is:

$$U_t^y = \mu P_t^{-\alpha} e_t^y + \tfrac{1}{1+\rho} \mu \tilde{P}_{t+1}^{-\alpha} \tilde{e}_{t+1}^o = \mu \times$$
$$\left(P_t^{-\alpha} \left(w_t + \chi_t R_t - \sigma \left(x_t, \mathbf{T}_t \right) \right) + \tfrac{1}{1+\rho} \tilde{P}_{t+1}^{-\alpha} \Big((1 - \tilde{\chi}_{t+1}) \tilde{R}_{t+1} + \tilde{\pi}_{t+1} + \tilde{\sigma}(x_{t+1}, \mathbf{T}_{t+1}) \Big) \right). \tag{12}$$

If a young person buys a unit of the factory today, costing σ_t, the loss in utility is $\mu P_t^{-\alpha} \sigma_t$. Purchase of one factory today increases expenditures next period by $\tilde{\pi}_{t+1} + \tilde{\sigma}_{t+1}$; the increase in the present value of utility next period due to the purchase of the factory is $\frac{1}{1+\rho} \mu \tilde{P}_{t+1}^{-\alpha} \left(\tilde{\pi}_{t+1} + \tilde{\sigma}_{t+1} \right)$. The equilibrium price-of-factory sequence requires that excess demand for the asset is 0, which, under rational expectation, requires satisfaction of the no-arbitrage condition

$$P_t^{-\alpha} \sigma_t = \frac{1}{1+\rho} P_{t+1}^{-\alpha} \left(\pi_{t+1} + \sigma_{t+1} \right). \tag{13}$$

Write this no-arbitrage condition, equation (13), as

$$\sigma_t = \frac{1}{1+\rho} \left(\frac{P_t}{P_{t+1}} \right)^\alpha \left(\pi_{t+1} + \sigma_{t+1} \right)$$

or

$$\sigma_{t+i} = \frac{1}{1+\rho} \left(\frac{P_{t+i}}{P_{t+1+i}} \right)^\alpha \left(\pi_{t+i+1} + \sigma_{t+i+1} \right),$$

so

$$\sigma_t = \frac{1}{1+\rho} \left(\frac{P_t}{P_{t+1}} \right)^\alpha \pi_{t+1} + \frac{1}{1+\rho} \left(\frac{P_t}{P_{t+1}} \right)^\alpha \left[\frac{1}{1+\rho} \left(\frac{P_{t+1}}{P_{t+2}} \right)^\alpha \left(\pi_{t+2} + \sigma_{t+2} \right) \right].$$

By repeated substitution obtain

$$\sigma_t = \sum_{j=1}^{S} \left(\frac{1}{1+\rho} \right)^j \left[\left\{ \Pi_{s=0}^{j-1} \left(\frac{P_{t+s}}{P_{t+s+1}} \right)^\alpha \right\} \pi_{t+j} \right] + \left(\frac{1}{1+\rho} \right)^S \left[\left\{ \Pi_{s=0}^{S-1} \left(\frac{P_{t+s}}{P_{t+s+1}} \right)^\alpha \right\} \sigma_{t+S} \right]$$

If the system converges to a steady state, then the second term goes to 0 as $S \to \infty$ and

$$\sigma_t = \sum_{j=1}^{\infty} \left(\frac{1}{1+\rho}\right)^j \left[\left\{\Pi_{s=0}^{j-1} \left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha\right\} \pi_{t+j}\right].$$

Note that

$$\Pi_{s=0}^{j-1} \left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha = \left(\frac{P_t}{P_{t+j}}\right)^\alpha$$

Using this relation we have

$$\sigma_t = P_t^\alpha \sum_{i=1}^{\infty} (1+\rho)^{-i} P_{t+i}^{-\alpha} \pi_{t+i}, \tag{14}$$

π is independent of the stock and, for fixed T, constant. Under this condition the expression for the asset price reduces to

$$\sigma_t = \pi P_t^\alpha \sum_{j=1}^{\infty} \left(\frac{1}{1+\rho}\right)^j P_{t+j}^{-\alpha}. \tag{15}$$

■

A.3 Proof of Corollary 1

Proof. (i) The imposition of the no-arbitrage condition simplifies the lifetime welfare expression of the young, equation (12), to:

$$U_t^y = \mu \left[p(T_t, x_t)^{-\alpha}(w(T_t) + \chi_t R(T_t)) + \frac{p(T_{t+1}, x_{t+1})^{-\alpha}}{1+\rho}(1-\chi_{t+1})R(T_{t+1})\right]. \tag{16}$$

The no-arbitrage condition implies that the young generation's utility is independent of the asset price. A loss in utility from the higher asset price in the first period equals the discounted utility gain from increased profits and asset price in the second period. As a consequence, the young generation's expenditure equals wage income in the first period and their share of the tax revenue in the first and second period. Their welfare considerations are limited to these expenditure components and the price effects.

(ii) The same holds for all future generations. Asset prices enter only the welfare expression of the current old generation. The current owners of the asset capture all future benefits reflected in a changed asset price. ■

A.4 Proof of Proposition 3

Proof. Using equation (3), and the definitions of ς and x_∞ in equations (4) and (5), the BAU trajectory is monotonic if and only if the initial condition is less than or equal to the root of $(1+\bar{r}(0,x))\,x = x_\infty$, which is equivalent to $x_0 \le C\frac{\varsigma-1}{r}$, or $x_0 \le \frac{1}{\varsigma-1}x_\infty$.

The old generation's remaining lifetime welfare consists of the utility it obtains from current consumption,

$$U_t^o(\varepsilon) = \mu\left(p\left(x_t, T_t\right)^{-\alpha}(1-\chi)R_t + \sum_{i=0}^{\infty}(1+\rho)^{-i}p\left(x_{t+i}, T_{t+i}\right)^{-\alpha}\pi_{t+i}\right). \tag{17}$$

We start with the derivative of the second term in U^o, the return to holding the asset. We differentiate each term in the sum by $T_i = \varepsilon\bar{T}_i$, recognizing that T_i has a direct effect on $\pi_{t+i}p_{t+i}^{-\alpha}$ and an indirect effect, via its effect on x_{t+j}, on $\pi_{t+j}p_{t+j}^{-\alpha}$ for $j > i$. We use $T_i = \varepsilon\bar{T}_i$, so $dT_i = \bar{T}_i d\varepsilon$.

$$\begin{aligned}
\frac{\mathrm{d}\sum_{i=0}^{\infty}(1+\rho)^{-i}\pi_i p_{t+i}^{-\alpha}}{\mathrm{d}\varepsilon} &= \frac{\partial \pi_t p_t^{-\alpha}}{\partial T_t}\bar{T}_t \\
&+(1+\rho)^{-1}\left[\frac{\partial \pi_{t+1}p_{t+1}^{-\alpha}}{\partial T_{t+1}}\bar{T}_{t+1} + \frac{\partial \pi_{t+1}p_{t+1}^{-\alpha}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial T_t}\bar{T}_t\right] \\
&+(1+\rho)^{-2}\left[\frac{\partial \pi_{t+2}p_{t+2}^{-\alpha}}{\partial T_{t+2}}\bar{T}_{t+2} + \frac{\partial \pi_{t+2}p_{t+2}^{-\alpha}}{\partial x_{t+2}}\left(\frac{\partial x_{t+2}}{\partial T_{t+1}}\bar{T}_{t+1} + \frac{\partial x_{t+2}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial T_t}\bar{T}_t\right)\right] \\
&+(1+\rho)^{-3}\left[\frac{\partial \pi_{t+3}p_{t+3}^{-\alpha}}{\partial T_{t+3}}\bar{T}_{t+3} + \frac{\partial \pi_{t+3}p_{t+3}^{-\alpha}}{\partial x_{t+3}}\left(\frac{\partial x_{t+3}}{\partial T_{t+2}}\bar{T}_{t+2} + \frac{\partial x_{t+3}}{\partial x_{t+2}}\frac{\partial x_{t+2}}{\partial T_{t+1}}\bar{T}_{t+1} + \frac{\partial x_{t+3}}{\partial x_{t+2}}\frac{\partial x_{t+2}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial T_t}\bar{T}_t\right)\right] \\
&+....
\end{aligned}$$

We simplify this expression using the fact that at $\varepsilon = 0$, $T_0 = T_1 = ... = 0$. Evaluating the different expressions along the BAU trajectory, we have

$$\frac{\partial \pi_i p_i^{-\alpha}}{\partial T_i} = 0; \quad \pi_0 = \pi_1 = ... = \pi; \quad \text{and } \frac{\partial \pi_i p_i^{-\alpha}}{\partial x_i} = \eta x_i^{\alpha-1}, \text{ with } \eta \equiv \alpha\pi\left(\frac{w}{\gamma}\right)^{-\alpha} > 0.$$

Using the convention that $\prod_{j}^{j-1} z_j = 1$, we write the i'th term in the sum above as $(1+\rho)^{-i}\theta_i$, with

$$\theta_i \equiv \eta x_{t+i}^{\alpha-1}\left[\sum_{j=0}^{i-1}\left\{\frac{\partial x_{t+i-j}}{\partial T_{t+i-j-1}}\bar{T}_{t+i-j-1}\left(\prod_{k=0}^{j-1}\frac{\partial x_{t+i-k}}{\partial x_{t+i-k-1}}\right)\right\}\right].$$

The initial condition is x_t and the BAU steady state is x_∞. The assumption that $x_t < \frac{1}{2}\frac{\varsigma}{\varsigma-1}x_\infty$, with $\frac{1}{2}\frac{\varsigma}{\varsigma-1} > 1$ by inequality (4), implies that $\left(\prod_{k=0}^{j-1}\frac{\partial x_{t+i-k}}{\partial x_{t+i-k-1}}\right) > 0$. By assumption, $\tilde{T}_{t+i-j-1} \geq 0$ with strict inequality for some $i-j-1 \geq 0$, and we have $\frac{d\bar{r}_t}{dT_t} > 0 \Rightarrow \frac{\partial x_{t+1}}{\partial T_t} > 0$. Consequently $\theta_i \geq 0$ with strict inequality holding for some i.

The old also receive a share of the tax revenue. The effect of a tax increase on current tax revenue is

$$\begin{aligned}\left.\frac{dP^{-\alpha}(1-\chi)R}{d\varepsilon}\right|_{\varepsilon=0} &= \underbrace{(1-\chi)R\frac{dP^{-\alpha}}{d\varepsilon}}_{=0} + (1-\chi)P^{-\alpha}\left.\frac{dR}{d\varepsilon}\right|_{\varepsilon=0} \\ &= (1-\chi)P^{-\alpha}\frac{\alpha\left(1+\frac{\alpha}{\beta(1-\alpha)}\right)^{-\beta}}{1-\alpha}\bar{T}_t > 0 \qquad (18)\end{aligned}$$

Given that the derivatives of both terms in U^o are positive, a small tax trajectory increases the welfare of the old generation. For a positive current tax, $R_t > 0$, and the old generation's utility strictly increases in its share of the tax revenue. ■

A.5 Proof of Proposition 4

Proof. The lifetime welfare of the young, from equation (16), is

$$U_t^y(\varepsilon) = \mu p(\bar{T}_t\varepsilon, x_t)^{-\alpha}\left(w(\bar{T}_t\varepsilon) + \chi R(\bar{T}_t\varepsilon) + \frac{\left(1+\bar{r}(\bar{T}_t\varepsilon, x_t)\right)^\alpha}{1+\rho}(1-\chi)R(\bar{T}_{t+1}\varepsilon)\right).$$

Differentiating this expression with respect to ε gives

$$\frac{dU_t^y}{d\varepsilon} = \frac{d}{d\varepsilon}\mu P_t^{-\alpha}(w(\bar{T}_t\varepsilon)+\chi R(\bar{T}_t\varepsilon)) + \frac{d}{d\varepsilon}\left[\mu P_t^{-\alpha}\frac{\left(1+\bar{r}(\bar{T}_t\varepsilon, x_t)\right)^\alpha}{1+\rho}(1-\chi)R(\bar{T}_{t+1}\varepsilon)\right]$$

We know that the first order effect of a tax introduction on output measured in utils is zero simply because the pre-tax allocation maximizes current aggregate utility. Given the constancy of shares in the Cobb-Douglas production function in manufacturing, the two remaining components of income, w and R, also have to add up to zero: $\left.\frac{dP^{-\alpha}w}{d\varepsilon}\right|_{\varepsilon=0} + \left.\frac{dP^{-\alpha}R}{d\varepsilon}\right|_{\varepsilon=0} = 0$. Using the fact that $R(0) = 0$ such that $\left.\frac{dP^{-\alpha}R}{d\varepsilon}\right|_{\varepsilon=0} = P^{-\alpha}\left.\frac{dR}{d\varepsilon}\right|_{\varepsilon=0}$ and the assumption that the first two tax rates are equal, the expression simplifies to

$$\begin{aligned} \left.\frac{dU_t^y}{d\varepsilon}\right|_{\varepsilon=0} &= \mu\left[P_t^{-\alpha}(-1+\chi)\,\frac{dR}{d\varepsilon} + P_t^{-\alpha}(1-\chi)\frac{(1+\bar{r}(\bar{T}_t\varepsilon, x_t))^{\alpha}}{(1+\rho)}\,\frac{dR}{d\varepsilon}\right]_{\varepsilon=0} \\ &= \mu P_t^{-\alpha}(1-\chi)\left(\frac{(1+\bar{r}(0,x_t))^{\alpha}}{1+\rho} - 1\right)\left.\frac{dR}{d\varepsilon}\right|_{\varepsilon=0} \end{aligned}$$

The young generation loses income $-\,P_t^{-\alpha}\frac{dR}{d\varepsilon}$ in the first period through an increase in the tax, but is able to recuperate $\chi P_t^{-\alpha}\frac{dR}{d\varepsilon}$ in the form of tax revenues. It gains $P_t^{-\alpha}(1-\chi)\frac{(1+\bar{r})}{(1+\rho)}\,\frac{dR}{d\varepsilon}$ in the next period. The first-order response of tax revenue to a small tax introduction is positive: $\left.\frac{dR(\bar{T}_t,\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = \frac{\alpha}{1-\alpha}\left(1+\frac{\alpha}{\beta(1-\alpha)}\right)^{-\beta}\bar{T}_t > 0$. Under the assumption that $\bar{T}_0 = \bar{T}_1$, we establish the following condition under which a small positive tax increases the initial young agent's *lifetime* welfare (16):

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{(1+\bar{r}(0,x_0))^{\alpha}}{1+\rho} - 1\right)\bar{T}_0 > 0. \tag{19}$$

With $\chi \in [0,1]$, a small tax increases the lifetime welfare of the young generation if and only if $\chi < 1$ and $(1+\bar{r}(0,x_t))^{\alpha} > (1+\rho)$. A small tax creates a zero first order welfare effect for the young generation that receives all tax revenue ($\chi = 1$). Condition (b) in the Proposition is equivalent to $\bar{r}(0,x_t) > (1+\rho)^{\frac{1}{\alpha}} - 1$. For $\rho > 0$, the expression on the right side of the previous inequality is positive. Thus, a necessary condition for the young to benefit from a tax is that the resource is below its 0-tax steady state, and is in the process of sufficiently strong recovery. ■

A.6 Proof of Proposition 5

Proof. The last equation in system (1) implies that $p(T_t, x_{t+1})^{-\alpha} = p(T_t, x_t)^{-\alpha}(1+\bar{r}(T_t, x_t))^{\alpha}$. This equality and the fact that the young generation's welfare is linear in χ, from equation (16), implies that

$$\frac{dU_0^y}{d\chi} < 0 \Leftrightarrow \left(\frac{(1+\bar{r}(T_0,x_0))^{\alpha}}{1+\rho} - 1\right)\bar{T}_0 > 0. \tag{20}$$

We also have $\frac{d\bar{r}_t}{dT_t} > 0$. This inequality and inequalities (19) and (20) imply that if the young benefit from a small tax, then they prefer to receive all of the tax revenue when they are old, i.e. they prefer $\chi = 0$. In contrast, if the young are harmed by a small tax, then provided that the tax is small they prefer to receive all of the tax revenue when young ($\chi = 1$). ■

A.7 Proof of Proposition 6

Proof. With ξ the share of the old generation's tax revenue transferred to the young in the period when the tax is first imposed, the first period's tax receipts are now $(\chi_0 + (1-\chi_0)\xi)R_0$ for the young and $(1-\chi_0)(1-\xi)R_0$ for the old. Under the assumption that current and next period tax rates are changed by the same small amount and that χ is constant, an argument that parallels the derivation in Appendix A.5 leads to the following condition for the young to benefit from the combined transfer and tax:

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{(1+\bar{r}(0,x_0))^{\alpha}}{1+\rho} - (1-\xi)\right)\bar{T}_0 > 0. \qquad (21)$$

Setting $\xi = 0$, equation (21) reproduces equation (19). For

$$\xi > \xi^{crit} \equiv 1 - \frac{(1+\bar{r}(0,x_0))^{\alpha}}{1+\rho}$$

the young strictly prefer the combined tax and transfer compared to the BAU status quo. Even if the resource is degrading on the BAU trajectory, $\xi^{crit} < 1$. Therefore, by transferring less than their entire share of the tax revenue to the young, the old can make the young better off under a small tax. Because Proposition 3 states that the tax improves the old generation's welfare even if they receive none of the tax revenue, the old are obviously better off under the combined tax and transfer, compared to the status quo. ■

A.8 Proof of Proposition 7

Proof. Using a derivation parallel to that contained in Appendix A.5, we have

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{e^{-(a-b)\alpha}\,(1+\bar{r}(0,x_0))^{\alpha}}{1+\rho} - 1\right)\bar{T}_0 > 0.$$

The second inequality is equivalent to

$$\left(\frac{1+\bar{r}(0,x_0)}{e^{(a-b)}}\right)^{\alpha} > 1+\rho. \tag{22}$$

The left side of inequality (22) is decreasing in $a-b$, so an increase in $a-b$ decreases the set of parameter values and initial conditions under which the inequality is satisfied, i.e. the circumstances under which the young benefit from the tax. ■

B Appendix for Referee

This appendix collects information not intended to be published.

B.1 Future generations

Merely in order to avoid uninteresting complications, we assume that for future generations the tax is constant: $\bar{T}_0 = \bar{T}_1 = \bar{T}_2 ....$ The life-time welfare of the next young generation is

$$U_1^y(\varepsilon) = \mu p(\bar{T}_1\varepsilon, x_1)^{-\alpha}\left(w(\bar{T}_1\varepsilon) + \chi R(\bar{T}_1\varepsilon) + \frac{\left(1+\bar{r}(\bar{T}_1\varepsilon, x_1)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_2\varepsilon)\right).$$

Differentiating this expression with respect to ε gives

$$\frac{dU_1^y}{d\varepsilon} = \frac{d}{d\varepsilon}\mu P_1^{-\alpha}(w(\bar{T}_1\varepsilon)+\chi R(\bar{T}_1\varepsilon)) + \frac{d}{d\varepsilon}\left[\mu P_1^{-\alpha}\frac{\left(1+\bar{r}(\bar{T}_1\varepsilon, x_1)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_2\varepsilon)\right]$$

Using the simplifications of Appendix A.5, especially the fact that $R(0) = 0$, and the fact that $\frac{\partial P_1^{-\alpha}}{\partial x_1} = \alpha P_1^{-\alpha}x_1^{-1}$, the expression simplifies to

$$\begin{aligned}
\left.\frac{dU_1^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \;&\Leftrightarrow\; \bar{T}_1 P_1^{-\alpha}(1-\chi)\left(\frac{(1+\bar{r}(0,x_1))^{\alpha}}{1+\rho} - 1\right)\left.\frac{dR}{d\varepsilon}\right|_{\varepsilon=0} > -\bar{T}_0 w(0)\frac{\partial P_1^{-\alpha}}{\partial x_1}\frac{\partial x_1}{\partial T_0} \\
&\Leftrightarrow\; (1-\chi)\left(\frac{(1+\bar{r}(0,x_1))^{\alpha}}{1+\rho} - 1\right)\bar{T}_0 > -\left(w(0)\alpha x_1^{-1}\frac{\partial x_1}{\partial T_0}\right)\left(\frac{1}{dR/d\varepsilon|_{\varepsilon=0}}\right)\bar{T}_0
\end{aligned}$$

Comparing this condition to inequality (19), we see that when the stock is degrading (i.e. $\bar{r}(0, x_0) < 0$), a small tax is more likely to benefit the next period's young generation compared to today's, which always loses in the absence of transfers. The difference arises for two reasons: A lower stock increases the BAU growth rate, $\frac{d\bar{r}(0,x_t)}{dx_t} = -r < 0$, so that the left side is less negative. The right side of the inequality above is negative. Therefore, the condition here is weaker than the condition in inequality (19). In fact, it is satisfied for any initial stock value in the calibration used in Section 8.

B.2 Numerical Method

We approximate $\Upsilon(x_{t+1})$ and $\bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1})) \equiv \Phi(x_{t+1})$ as polynomials in x_{t+1}, and find coefficients of those polynomials so that the solution to

$$\max_{T_t} \mu P^{-\alpha}(x_t, T_t) Y(T_t) + \frac{1}{1+\rho}\mu \left\{P^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(\Upsilon(x_{t+1})) + \Phi(x_{t+1}) + R(\Upsilon(x_{t+1}))\right\}$$

subject to equation (10) approximately equals $\Upsilon(x_t)$. We use 13-degree Chebyshev polynomials evaluated at 13 Chebyshev nodes on the $[0.1, 0.9]$ interval. At each node the following conditions have to be approximately satisfied

$$\begin{aligned} \Phi(x_t) &= \frac{1}{1+\rho}\left\{p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(\Upsilon(x_{t+1})) + \Phi(x_{t+1}) + R(\Upsilon(x_{t+1}))\right\} \\ &\frac{d}{dT_t}\left[\mu P^{-\alpha}(x_t, T_t) Y(T_t) + \frac{1}{1+\rho}\Omega\right] = 0 \\ \text{with} \quad \Omega &\equiv \mu\left\{P^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(\Upsilon(x_{t+1})) + \Phi(x_{t+1}) + R(\Upsilon(x_{t+1}))\right\} \end{aligned} \tag{23}$$

subject to $x_{t+1} = (1 + \bar{r}_t(x_t, T_t))\, x_t$ and $T_t = \Upsilon(x_t)$. Each node gives two non-linear equations in the coefficients of the two polynomials. If the number of nodes equals the degree of approximation (i.e. the number of coefficients of each polynomial), the system of non-linear equations can be solved using a root-finding method. We employ Mathmatica's FindRoot command which solves the system in less than a minute on a standard personal computer. We increased the number of nodes and degree of approximation to 16 in the social planner's problem to arrive at satisfactory levels of accuracy.

Evaluating the equation system (23) using the solution approximations gives a statistic for the goodness of fit. The figures 5 and 6 illustrate that the residual errors are 5 orders of magnitudes below the solution values.

In the text we presented the solution for $\chi = 0$ for reasons explained in Section 6. Here we discuss the $\chi = 1$ case, where in each period the young receives all of the tax revenue. This change reduces the incentive for generations to preserve the resource and consequently lower tax rates are chosen at all levels of the stock, x. Figure 7 plots the policy function for the efficient bargaining problem for $\chi = 0$ (solid) and $\chi = 1$ (dotted). At its maximal difference, the $\chi = 0$ policy function lies 20% below the case reported in the text. This considerable difference in the policy function,

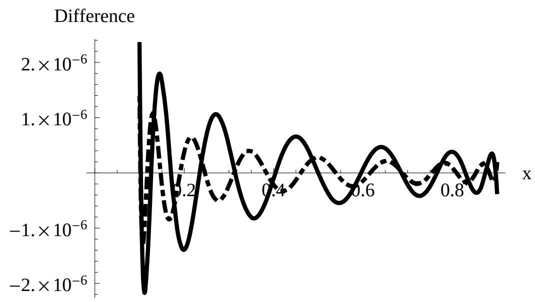

Figure 5: Deviation of asset price approximation from true value outside of approximation nodes for the efficient bargaining (solid) and the social planner's (dot-dashed) problems

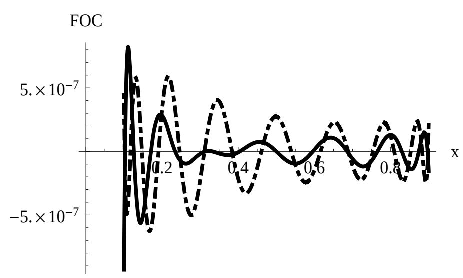

Figure 6: Deviation of policy function approximation from true value outside of approximation nodes for the efficient bargaining (solid) and the social planner's (dot-dashed) problems

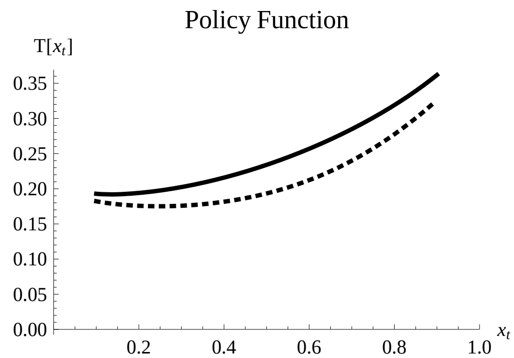

Figure 7: Policy Function for $\chi = 0$ (solid) and $\chi = 1$ (dotted).

however, has little impact on the value function or the transition equation of the stock variable. The steady state under the less conservative tax policy is at 0.38 which is only 5% under the $\chi = 0$ equilibrium level.