The Transmission of Risk in the Energy Supply Chain

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ABSTRACT

We present a model of the transmission of risk in a supply chain. Starting with exogenous processes for the net supply of the upstream input and the demand for the downstream output we construct the equilibrium process for the price of the output when the capital asset that transforms the input into the output faces operational constraints. We present comparative statics for the spread of prices between the input and the output when the market for the capital asset is either perfectly competitive or a monopoly. We calibrate our model for the case of crude oil and gasoline. Our results can be applied in valuing oil refineries, as well as to determine the optimal amount to hedge, when only limited hedging is possible.

Introduction

In their most abstract formulation, supply chains consist of three components: an upstream primary input; a downstream output product; and a capital asset, the employment of which turns the input into the output. One can give numerous examples: crude oil is transformed into heating oil and gasoline at a refinery; bauxite ore is transformed into aluminum in aluminum smelters; raw tomato and potato are converted into paste and chips; jet fuel is transformed into air travel using an airplane. In each case, characteristics of the supply of the input, the demand of the output, and the supply chain process combine to characterize the behavior of the spread in prices between the input and output. We offer a general equilibrium model for the spread, based on realistic assumptions on the three components of supply chains.

Much of the intuition of our model is captured in an example discussed in the chapter XI of *Politics*, by Aristotle. Aristotle describes a transaction that proved particular gainful for one of the sages of antiquity, Thales of Miletus. It is told that Thales correctly forecasted a bountiful year for olive production in ancient Greece. Based on this prediction, months ahead of olive production, Thales made a down payment to secure the use of all the olive presses. When the large harvest materialized, with a large supply of olives, and a limited number of olive presses, Thales was able to rent out the olive presses for a large profit. Aristotle argues that Thales was able to make a profit by creating a monopoly. We point out an alternative explanation: assuming that the demand curve for olives and olive oil remain relatively unaffected by the production of olives, a year of extraordinary large production of olives would have several effects. Since olives are bountiful, the price of olive would drop; given the cheaper price of input, olive oil prices would also drop. However, once olive presses operate at capacity, the amount of olive oil available can no longer increase, and further drops in the price of the input are not reflected in the price of olive oil. At that point, the spread between olive prices and olive oil prices starts increasing as olive prices drop further. Widening spreads lead to profit for owners of olive presses, even if no one enjoys a monopoly.

The model we develop in this paper makes assumptions on the three components of the supply chain: we assume an exogenous random process for the net supply of the upstream input; an exogenous stochastic demand curve for the downstream output; and we assume constraints in the operation of the capital asset that transforms the input into the output. Our model allows us to make predictions on the dynamics of price of the output, including the spread between the input and the output, the volatility of the price of the output conditional on the price of the input, the correlation between the two prices, as well as a process for the value of the capital asset, and its volatility and correlations with the input and output prices. Assuming a link between the demand for the downstream product and the market return, our model can also produce conditional predictions for the beta coefficient between the returns of the output and the capital asset and the market portfolio.

Our theoretical model leads to several predictions: a) in a perfectly competitive market with no constraints in the operation of the capital asset, spreads converge to a fixed level. If the operation of the capital asset is constrained — either through capacity limits, or production adjustment costs — spreads deviate from this level. In a static model spreads are always positive, while a dynamic model allows for negative spreads in some periods; b) the profit accrue to profit asset is highly skewed; it is near zero for a significant fraction of time but jumps to a large positive level in certain periods. The capital asset does not earn smooth profit but benefits from these jumps. c) when demand is high enough, spreads under both a monopoly and a competitive market converge; d) very low input prices, and stable output demand increases spreads. In this case the capital asset operates almost at full capacity; e) the volatility of input and outprice factors increases the average spreads by increasing the frequency of producing at full capacity; f) correlation between input and output prices depends on the level of output demand and input price. When output demand is low and input price is high, the price of the input and the price of the output are highly correlated. When output demand is strong or input price is very low, the price of input and the price of the output diverge and the correlation drops significantly; g) the spread increases when the volatility of output demand increases. This happens because of convexity in the spread; h) correlation between input and output prices is higher in competitive markets than in monopolistic markets, and; i) output demand shocks result in bigger output price changes in a competitive market than in a monopolistic market.

In addition to developing an equilibrium model, we provide an empirical study of the model in the case where the input is crude oil, the output is gasoline, and the capital asset a refinery. Using the Simoulated Method of Moments and Kalman filtering approaches, we estimate the dynamics of gasoline demand under both the physical and risk-neutral measures using a time series of observed prices of gasoline and crude oil and by assuming that the gasoline market is competitive.

We also provide comparative statics that explore how other exogenous parameters, such as the demand for gasoline, the supply of oil and the production function, affect the endogenous variances and correlations. Since we have daily spot price data from 1984, we also examine differences in the observed behavior of oil and gasoline prices as well as the returns of refineries in different periods. In the early part of our sample the market for refined products was very competitive. Starting in the late 1990s there was substantial consolidation of refineries, and near the end of our sample period the market was much less competitive. The sample periods also differ in terms of the extent to which refineries were operating at capacity. In most of our sample period refineries were operating substantially below capacity. However, in the 2005-2007 period, production was much closer to capacity levels.

Our results illustrate the challenges associated with hedging input prices. Since in our model there are two state variables, in some situations the volatility of profits may not be meaningfully reduced by simply hedging input prices. Indeed, we show that small changes in parameters can lead to dramatic changes in the hedge ratio. It is possible that, due to the non-linearity of the relation between the downstream and upstream prices, non-linear hedging instruments, for example options, might provide a better hedge.

Crack Spread and Literature Review

The term "crack spread" refers to the difference between the price of certain derivatives of crude oil (mostly gasoline and heating oil) and the crude oil used to produce these refined products. The crack spread is generally believed to be a measure of refineries' profitability. There are synthetic contracts traded in NYMEX and other commodity exchanges which offer a 3-2-1 ratio, meaning that the value of the contract is the difference between the value of three units of crude oil and the sum of two units of gasoline and one unit of heating oil.[1] One of the goals of our paper is to understand the dynamics of the value of the crack spread in a structural, equilibrium, model.

A survey of several reports and articles by market analysts suggests that the market considers crack spreads as driven by mostly changes in the price of crude oil, and overlooks the effect of demand for gasoline as a potential second explanatory factor for crack spreads. Our model shows that crude oil price movements alone cannot explain the behavior of crack spreads. One needs to take into account the demand for gasoline. Furthermore, our model shows that the relationship between crude oil price and crack spread is not linear. Indeed, we show that when crude oil price and gasoline demand are correlated, the crack spread may have a U-shape relation with respect to crude oil price.

Our work differes from the current literature of gasoline price in several important ways. Part of the literature has focused on average volatility spillover from input to output markets (vertical spillover): Buguk, Hudson, and Hanson (2003) build an EGARCH model of catfish price volatility by incorporating the lagged error terms of price equation for the production inputs — corn and soy; Blair and Rezek (2007) evaluate the effect of Katrina on the cost passthrough of crude oil on gasoline. Using an ECM model, they show that in the period before, and long after the Katrina, every unit of increase in the price of crude oil results in

[1] Although such a contract can easily be replicated by making a portfolio of contracts on these three products, market participants prefer it because of its lower margin account requirements.

almost an increase of the same magnitude on gasoline prices. On the other hand, right after Katrina, a 10 cent increase in crude oil price, drives up the price of gasoline by 63 cents.

A key issue in modeling gasoline prices is the asymmetry between shocks to crude oil price and gasoline wholesale and retail prices. The asymmetry suggests that a positive shock to crude oil market transmits much quicker to gasoline prices than negative shocks. Borenstein and Shepard (2000) provides an explanation for why gasoline prices do not immediately react to the changes of crude oil price. Their model includes production adjustment costs and the possibility of storage. They use the response of gasoline futures prices to the innovations of crude oil futures with the same maturity. If the lag between gasoline and crude oil prices is due to the slow adjustment of production, then near-to-maturity contracts of gasoline should react only partially while long maturity contacts will react immediately with a full adjustment. Their results show that 1 cent increase in crude oil price eventually increases gasoline prices by 1.14 cents but this effect is 0.16 cents smaller for near-month futures contracts.

Radchenko (2004) examines whether the volatility of crude oil price affects the degree of asymmetric response of gasoline prices to oil price shocks. His findings are based on a VAR model and confirm that there is a negative relationship between volatility and the degree of price response asymmetry. He suggests that these findings may support a search model with Bayesian updating and oligopoly coordination in the gasoline market.

There is a large strand of the literature which focuses on the reduced-form behavior of underlying process and their effect of the price of spread derivatives. For example, Dempster, Medova, and Tang (2008) propose a two-factor model for the spread between two co-integrated assets. They directly model the spread by a mean reverting process and a stochastic mean-reverting long-run mean for that process. They use state-state representation and Kalman filtering to calibrate the model and then apply it for valuation of futures and options on spread between heating oil and WTI and also between WTI and Brent. In contrast, in our paper we build a structural model for modeling spreads.

The remainder of our paper is structured as follows. In Section I we present the structural model for the upstream and downstream markets of the supply chain as well as the model for the capital asset. Section II presents propositions that are consequences of our model. Section III calibrates our model for the case of refineries with crude oil as input, and gasoline as output. IV discusses potential applications of our model and concludes the paper.

I. Model

We develop a simple equilibrium model for the relationship between the price of the input and the output and the operational characteristics of the capital asset that transforms the input into the output. The model is built on assumptions on each of these three components. Throughout we consider the case of crude oil as input, gasoline and heating oil as output, and a refinery as the capital asset that transforms the input into the output.

Input: Crude Oil

The marginal cost of producing crude oil increases significantly with the agreggate level of production. We assume that the crude oil supply function follows a linear form:

$$P_C = \Delta + \alpha N Q_{US}$$

where Δ is a mean-reverting process which summarizes the shocks to global supply of oil and the non-US demand.[2] Crude oil supply is subject to random shocks with mean-reverting dynamics. Global crude oil supply is determined by the production of OPEC countries, large producers such as the United States and Russia, and a competitive fringe of small producers.

[2]In reality, there are two major sources of demand for crude oil in the global economy: U.S refinery demand (approximately 20% of global demand), consisting of gasoline, diesel and heating oil, and all non-US crude oil demand. In the short run, both demand factors are mean-reverting. Although both factors can potentially contain some seasonal factors, the literature and our empirical tests do not show a significant seasonal element in the price of crude oil.

The impact of OPEC trigger-type policies, where supplies are increased when prices cross a certain price level and decreased when prices drop below a different level, lead to crude oil prices that exhibit mean-reversion.

The net supply and non-US-Demand factor Δ follows a mean-reverting dynamics of

$$d\Delta = \mu_\Delta(\overline{\Delta} - \Delta)dt + \Delta\sigma_\Delta dW_\Delta$$

We assume that a unit of crude oil is converted to C_{CG} units of final gasoline product. Therefore, the global crude oil price can be expressed in terms of US gasoline production and exogenous non-US supply and demand shocks as

$$P_C = \Delta + \frac{\alpha}{C_{CG}} Q_{US}$$

The impact of endogenized crude oil price on the refinery sector's production decisions depends on the market power of US refinery industry in the crude oil market. If the refinery industry has monopoly power (monopsony) in procuring crude oil, then the production level Q_{US} in the price relationship will be treated as an object of optimization. If, on the other hand, the refinery industry behaves competitively, the price of crude oil will be treated exogenously in the refinery's optimization problem, even though, in equilibrium, the total US gasoline production will affect the global crude oil price.

Output: Gasoline

The price of gasoline is determined by an stochastic and seasonal demand factor X which depends on variables like income, weather and travel season and Q_{US}, the aggregate supply of gasoline to the market. The demand curve for gasoline in the United States will be assumed to be a linear function

$$P_G(q) = X - b_G Q_{US}$$

where b_G is a constant elasticity parameter. While a linear demand function is not perfectly realistic for gasoline, since it suggests the existence of a price above which demand for the gasoline is zero, we will use it as an approximation of the true demand function which allows us to derive analytical solutions.

With monthly factors M_t, we will assume that the logarithm of the deseasonalized gasoline demand X follows an Ornstein-Uhlenbeck mean-reverting process, resulting in demand that is log-normally distributed with mean and variance $(\mu_{X,t}, \sigma_{X,t})$. the log de-seasonalized process $x_t = ln(\frac{X_t}{M_t})$ follows the dynamics of

$$= \mu_x(\bar{x} - x)dt + x\sigma_x dw_x$$

Following the literature, we interpret the changes in demand function as a "mean" shock to the whole industry. The correlation between the logarithm of gasoline demand and the crude oil demand-supply (Δ) is given by $\rho = Cov(X, \Delta)$.

Capital Asset: US Refinery Industry

We assume that all the gasoline consumed in the US is produced domestically and there are no gasoline imports or exports. The domestic market is supplied by a refinery industry which converts crude oil into refined products including gasoline, heating oil and jet fuel. In reality, refineries may use a few different forms of crude oil — e.g. heavy crude and light sweet — depending on their engineering design and technical specification. For simplicity, we do not distinguish between different input types and treat crude oil as a homogeneous commodity. We focus on a horizon of approximately one year. In this horizon, the maximum refinery capacity is given and no capacity-building activity is feasible. Although refineries can make adjustments in their gasoline to distillate mix, historical data show that the ratio of gasoline to distillate production is approximately equal to 2. We abstract from the product-portfolio optimization problem of the refinery and simply assume an aggregate demand and product model.

The Cost of Gasoline Production

Production of gasoline involves explicit factor costs and implicit organizational and depreciation costs. The total cost $TC_G(q)$ function of gasoline production for a representative refinery consists of four major cost elements: $P_C(Q_{US}, \Delta)$ is the unit cost of crude oil which depends on the US demand and non-US demand and supply shocks. P_I is the cost of other inputs except crude oil which includes items such as energy, labor, maintenance and chemicals used in the process.

There are costs associated with producing at high levels of capacity utilization. These costs may occur because the refinery experiences higher probability of break-down in later periods (Chesnes (2009)), requires over-working its labor force and higher payments to contracts and also because the marginal units used to produce at high level might be more costly to build. We model the capacity-related costs using a quadratic form $\frac{\phi}{2}\frac{q^2}{\overline{q}}$, where ϕ is the coefficient and $\overline{q}$ is the maximum operational capacity. Finally, adjusting the production rate to a level different from average historical level is costly because it requires sudden changes of production plans and re-allocation of resources. Following Carlson, Khokher, and Titman (2007) we define

$$Z_t = \eta \int_0^t e^{\eta(u-t)} q_u du$$

as a weighted-average of historical production and $\frac{\delta}{2}(q_t - Z_t)^2$ as the adjustment cost with coefficient δ. The total cost function is given by

TC(q,Q$_{US},\Delta,Z) = P_I q + P_C(Q_{US},\Delta)\frac{q}{C_{CG}} + \frac{\phi q^2}{2\overline{q}} + \frac{\delta}{2}(q-Z)^2$

Optimization Problem

We assume that the total industry capacity is $\overline{q}$ which is given exogenously. We abstract from the random shocks to capacity due to natural disasters and industrial accidents. The industry can do costly adjustments in the operational capacity but can not produce above maximum capacity $\overline{q}$. The representative firm of the industry solves a dynamic optimization problem to maximize the value of the firm. The value function depends on three state variables: the historical average production Z, the level of gasoline demand X,and the supply in the global crude oil market.

$$\begin{aligned}
&Max_{q\leq\overline{q}}V(Z,X,\Delta) = E\int_0^\infty \left[P_G(X,Q_{US})q_t - P_I q_t - P_C(\Delta,Q_t)q_t - \phi\frac{q_t^2}{\overline{q}} - \frac{\delta}{2}(q_t - Z_t)^2\right]e^{-rt}dt \\
&dZ_t = \eta(q_t - Z_t)dt \\
&dx = \mu_x(\overline{x} - x_t) + x\sigma_x dW_x \\
&d\Delta = \mu_\Delta(\overline{\Delta} - \Delta) + \Delta\sigma_\Delta dW_\Delta
\end{aligned} \tag{1}$$

Using standard transformations, one gets the following Hamilton-Jacobi-Bellman (HJB) equations:

$$\begin{aligned}
&rV = Max_{q\leq\overline{q}}\Big[\pi(q) - P_I q - P_C q - \frac{\phi q}{2\overline{q}} + \delta(q-Z)V_Z + \mu_X(\overline{X} - X)V_X + \frac{1}{2}X^2\sigma_X^2 V_{XX} + \\
&\mu_\Delta(\overline{\Delta} - \Delta)V_\Delta + \frac{1}{2}\Delta^2\sigma_\Delta^2 V_{\Delta\Delta}
\end{aligned} \tag{2}$$

The first order conditions lead to

$$\frac{\partial P_G}{\partial Q_{US}}\frac{\partial Q_{US}}{\partial q}q + P_G - P_I - \frac{\partial P_C(Q)}{\partial Q}\frac{\partial Q_{US}}{\partial q}q - P_C(Q_{US},\Delta) - \delta V_Z - \phi\frac{q}{\overline{q}} - \lambda(q-\overline{q}) = 0 \tag{3}$$

where λ is the Lagrange multiplier associated with capacity constraints.

Market Structure

A priori, we do not know the exact market structure of the refinery industry. Nevertheless, from the empirical estimations one can conclude that the industry is not working under prefect competition assumptions. The spread between gasoline and crude oil changes significantly from one week to another while the cost of other inputs (wages, maintenance, etc) should be stable in such a time period. This suggests that the changes in the demand for gasoline and the price of crude oil play significant roles in explaining the dynamics of crack spreads. This is a fact we consider in our modeling. From a modeling perspective, an industry with market power or a competitive industry with adjustment or capacity-related costs can both arrive at the same conclusion regarding the effects of gasoline demand. As a result we present simples models of both type.

Competitive Refinery Sector

When production takes place in the interior ($q \leq \overline{q}$),

$$q = \frac{X - P_I - \frac{\Delta}{C_{CG}} - \delta V_Z}{b_G + \frac{\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}}} \tag{4}$$

If we assume that the adjustment costs are zero, $\delta = 0$, the optimal quantity and the price of gasoline are given by:

$$q_G^E = \min\left\{ \frac{X_G - \frac{\Delta}{C_{CG}} - P_I}{b_G + \frac{\phi}{\overline{q}} + \frac{\alpha}{C_{CG}^2}}, \overline{q} \right\}$$

$$P_G = \begin{cases} \frac{(\frac{\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}})X + b(\frac{\Delta}{C_{CG}} + P_I)}{b_G + \frac{\phi}{\overline{q}} + \frac{\alpha}{C_{CG}^2}} & \text{if } q^* \leq \overline{q} \\ \\ X - b_G\overline{q} & \text{Otherwise} \end{cases} \tag{5}$$

This result suggests that as long as there is some cost to produce at high capacity (i.e. $\phi > 0$), demand shocks will be relevant for the gasoline price even in a competitive market. Furthermore, there is a kink in the supply function of the refinery sector. When the domestic demand is too strong compared to net global oil demand-supply, the industry works close or at the full capacity and the production rate is independent of crude oil market situation.

The price of the input factor — crude oil in our case — is determined through the feedback of the refinery demand. As empirical evidence (Kilian (2009)) also points out, the last equation implies that when US demand is high and refineries work at maximum capacity, US demand shocks do not influence crude oil price. It is therefore expected that in the binding region, the co-movement between US gasoline prices and global crude oil prices be lower than the interior.

$$P_C = \Delta + \alpha(\min\left\{\frac{X_G - \frac{\Delta}{C_{CG}} - P_I}{b_G + \frac{\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}}}, \overline{q}\right\})$$

$$P_C = \begin{cases} \frac{(b_G + \frac{\phi}{\overline{q}} + \frac{\alpha}{C_{CG}^2} - \frac{\alpha}{C_{CG}})\Delta - \alpha(X_G - P_I)}{b_G + \frac{\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}}} & \text{if } q_G^E \leq \overline{q} \\ \\ \Delta + \alpha\overline{q} & \text{Otherwise} \end{cases} \tag{6}$$

Crack spreads will be given by

$$CS = \frac{(\frac{b_G}{C_{CG}} - b_G - \frac{\phi}{\overline{q}} - \frac{\alpha}{C_{CG}^2} + \frac{\alpha}{C_{CG}})\Delta + (\alpha + \frac{\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}})X_G + (b_G + \alpha)P_I}{b_G + \frac{\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}}} \tag{7}$$

Refinery Sector with Market Power

To provide a benchmark, we calculate the equilibrium quantity and prices for a refinery industry with market power. Such an industry considers the effect of its production decisions on the equilibrium prices in the global oil market.

$$q^{GE} = \min\left\{\frac{X_G - \frac{\Delta}{C_{CG}} - P_I}{2b_G + \frac{2\alpha}{C_{CG}} + \frac{\phi}{\overline{q}}}, \overline{q}\right\} \tag{8}$$

The price of gasoline is given by:

$$P_G = \begin{cases} \frac{(b + \frac{2\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}})X + b(P_I + \frac{\Delta}{C_{CG}})}{2b_G + \frac{2\alpha}{C_{CG}} + \frac{\phi}{\overline{q}}} & \text{if } q^* \leq \overline{q} \\ \\ X - b_G\overline{q} & \text{Otherwise} \end{cases} \tag{9}$$

The crude oil price is determined by

$$P_C = \Delta + \alpha(\min\left\{\frac{X_G - \frac{\Delta}{C_{CG}} - P_I}{2b_G + \frac{2\alpha}{C_{CG}} + \frac{\phi}{\overline{q}}}, \overline{q}\right\})$$

$$P_C = \begin{cases} \frac{(2b_G + \frac{\phi}{\overline{q}} + \frac{2\alpha}{C_{CG}^2} - \frac{b}{C_{CG}})\Delta - \alpha(X_G - P_I)}{2b_G + \frac{2\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}}} & \text{if } q_G^E \leq \overline{q} \\ \Delta + \alpha\overline{q} & \text{Otherwise} \end{cases} \tag{10}$$

Crack spreads will be given by

$$CS_{Monopoly} = \begin{cases} CS = \frac{(\frac{2b_G}{C_{CG}} - 2b_G - \frac{\phi}{\overline{q}} - \frac{2\alpha}{C_{CG}^2} + \frac{1}{C_{CG}})\Delta + (2\alpha + \frac{1}{C_{CG}^2} + \frac{\phi}{\overline{q}})X_G - (2b_G + \alpha)P_I}{2b_G + \frac{2\alpha}{C_{CG}^2} + \frac{\phi}{\overline{q}}} & \text{if } q^* \leq \overline{q} \\ X - (b + \alpha)\overline{Q}_{US} - \Delta & \text{Otherwise} \end{cases} \tag{11}$$

II. Comparative Statics

Observation 1: Higher demand for gasoline and higher crude oil price volatility increases the expected spreads. The effect is stronger when the level of mean demand for gasoline is higher.

In a competitive market the price of output is equal to input price as long as the capacity constraint is not binding. When the capacity binds, the demand shocks become relevant and the difference between price output and input increases.

A time-varying volatility regime for gasoline demand and crude oil price is implemented. Figure 1 shows the effect of gasoline demand volatility on crack spreads. In the low volatility regime, the probability of hitting the capacity constraint is low and therefore the crack spread is smaller. On the other hand, in the high volatility regime the price of crude oil and gasoline are more disconnected and crack spread increases. We notice that the crack spread under competitive market is systematically smaller than monopoly.

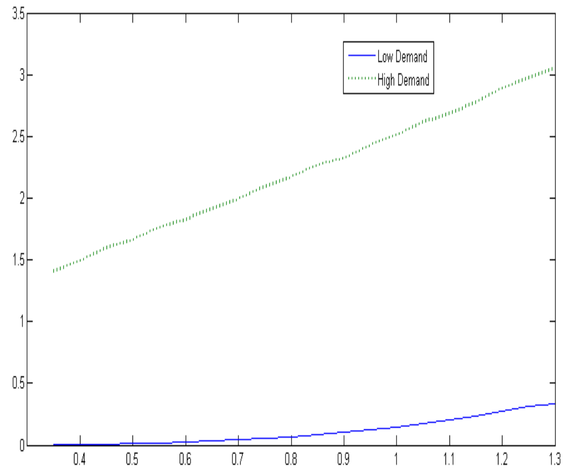

Figure 1. Effect of Demand Volatility on Crack Spreads

Observation 2: With a fixed demand parameter, decreasing input price (crude oil in this case) will increase crack spreads after some point.

Intuitively, when crude oil price is low gasoline production increases and therefore the probability of meeting capacity constraints goes up. The model suggests that when crude oil price is very low, crack spreads increase quickly because gasoline price can not be lower than a certain amount dictated by capacity constraints. This result can be empirically tested in a regression of $\text{Gasoline}/\text{Crude} = a + b\text{Crude}$ which can also be written as $\text{Gasoline} = a\text{Crude} + b\text{Crude}^2$.

Observation 3: The probability of hitting production boundary has a monotonic negative relationship with the covariance between gasoline demand and crude oil price shocks.

From the genertic relationship $q_G^E = \frac{m_1 X - m_2 \Delta - K}{\beta}$ (m_1,m_2,β,K are coefficients given by market structure and other parameters) one can see that when output and input shocks are highly

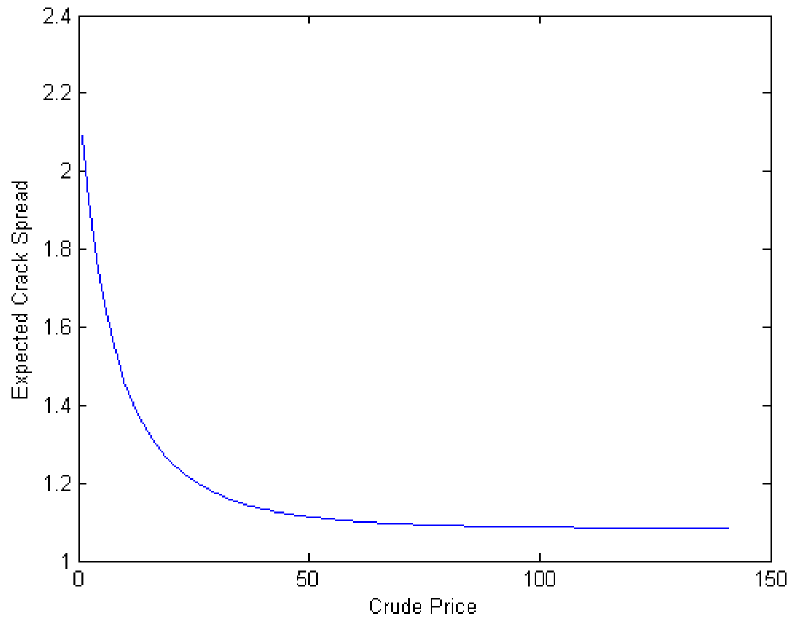

Figure 2. Crack Spreads vs Crude Price

correlated, the optimal production is mostly in the interior. The neccessary condition for $q_G^E > \overline{q}$ is that X has a high realization while Δ is low.

Observation 4: Gasoline price and crack spreads volatility follow a regime-switching behavior.

When production takes place in the interior, crack spreads are small and shocks propogate from one market to other one. Whereas, in the boundary regions shocks remain in the same market where they are originated and hence the volatility regime changes significantly.

III. Model Calibration and Estimation of Parameters

A. Objectives and Data

In a competitive market, there is not much to estimate regarding the spreads between input and output. Dynamic spread is generated when there are some deviations from perfect competition. As pointed out before, we do not know the true economic model of refinery industry. However, it was shown that if one assumes a major type of friction (market power, capacity constraints, capacity costs, adjustment costs) one arrives at a similar functional form for the relationship between gasoline and crude. In all of these models gasoline price is determined by some fuction of crude price and (unobserved) demand shock. A full specification of relationship between two prices requires assumptions regarding the functional form of demand process, too. With a linear demand, one can propose a general reduced form of $P_G = d + m_1 P_C + m_2 X$ where $\{d, m_1, m_2\}$ are unknown coefficients to be estimated. In a perfectly competitive market $m_2 = 0$.

We calibrate our simple model to gain some intuition over the magnitude and dynamics of the key variables. The input of calibration process consists of observable variables (gasoline price, crude price, gasoline production or $\{P_G, P_C, q\}$) and the goal is to estimate the values for structural parameters ($\{m_1, m_2, P_I\}$) as well as the dynamics of demand process ($\{\mu_X, \overline{X}, \sigma_X\}$) ideally under both physical and risk-neutral measures.

Our data set consists of weekly observation of crude and gasoline price and gasoline production from 1990/11/02 to 2010/04/23 (1017 observations in total), obtained from EIA website. For futures prices we use Bloomberg data for contracts traded in NYMEX between 2005/10/07 and 2009/09/25 (218 observations).

A.1. Transition, Measurement and Updating Equations

Using standard methods of state-space representation and leting $Z_t = P_G - m_1 P_C$ or $Z_t = P_G - m_1 q$, the transition and measurement equations of Kalman filter is defined as:

$$
\begin{aligned}
&X_{t+1} = c_X + HX_t + \varepsilon_1 \\
&Z_t = FX_t + d\varepsilon_2 \\
&c_X = \mu_X \overline{X}, H = (1 - \mu_X) \\
&F = m_2 \\
&\mathbb{E}(\varepsilon_1) = 0 \\
&\mathbb{E}(\varepsilon_2) = 0 \\
&\sigma_T = \mathbb{E}(\varepsilon_1^2) \\
&\sigma_M = \mathbb{E}(\varepsilon_2^2) \\
&< \varepsilon_1, \varepsilon_2 >= 0;
\end{aligned}
\tag{12}
$$

The prediction equation and estimated covariance used for filtering is given by

$$\hat{X}_{t+1|t} = \mathbb{E}(X_{t+1}|Z_t) = c_X + H\hat{X}_t + A_{t+1}(Z_{t+1} - \hat{Z}_{t+1})$$

$$\begin{aligned}
\hat{Z}_t &= F(c_X + H\hat{X}_t) + d \\
A_t &= R_t F_t Q_t^{-1} \\
R_t &= H\Sigma_{t-1}H + \sigma_T \\
\Sigma_t &= R_t - A_t Q_t A_t^{'} \\
Q_t &= (F_t R_t F_t + \sigma_M)
\end{aligned} \tag{13}$$

To estimate the unkown parameters the likelihood function of the Kalman filter should be maximised. Since the errors are normally distributed, the log likelihood function can be calculated using each period's estimated variance R_t and the error $e_t = Z_t - F\hat{X}_{t|t-1}$

$$\text{L}(\theta, P_0) = \sum_{t=2}^{t=T} -\tfrac{1}{2}ln(R_t) - \tfrac{1}{2}e_t R_t^{-1} e_t$$

Where P_0 is the prior regarding the mean and variance of X_0. The parameters can be found by minimizing $L(\theta, P_0)$ using numerical procedures in Matlab.

A.2. Estimation of Parameters Under Physical Measure

The unkown parameter set of our problem consists of $\theta = [m_1, m_2, d, X, \mu_X, X_0]$. We run Kalman filter for two different specifications:

1) Price Model: The first specification is the reduced-form relation between gasoline and crude prices. To include the cost of other inputs P_I into the estimation, we assume that $P_I = \alpha P_C + d$. Part of this cost element is proportional to crude price and is captured by $(1+\alpha)P_C$ term. The remaining part appears as a fixed term in the estimation equa-

m_1	m_2	d	μ	$\overline{X}$	X_0
1	3.55	-0.58	0.09	1.86	1.95

Table 1
Parameter Estimation for the Entire Period

tion. The economic model finally reduces to the following equation to be estimated: $P_G = m_1 P_C + m_2 X + d + \varepsilon_1, \varepsilon_1 = N(0, \sigma_M)$, where P_G and P_C are observable, X unobservable state variable and $\{m_1, m_2, d, X_0, \mu_X, \overline{X}\}$ are parameters to be estimated.

The Kalman filter is first applied for the entire sample between 1990-2010 (1017 observations). The estimated parameters are reported in table 1. Graph 3 shows the filtered values of unobservable variable X. From the filtered values we estimate the variance of demand process, $\sigma_X = 1.7$.

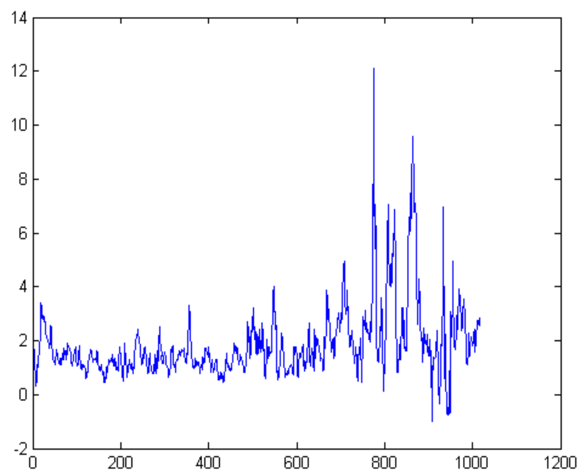

Figure 3. Estimated Values of Demand Process (X), 1990-2010

The procodure is repeated this time for each year between 1990-2010. The estimated parameters for annual estimations are reported in table 2

Year	m_1	m_2	d	μ	$\overline{X}$	X_0
1991	0.62	1.40	-0.040	0.053	9.69	9.50
1992	1.01	0.83	-0.64	0.19	5.51	3.96
1993	1.26	0.69	-3.20	0.085	2.19	1.77
1994	0.94	1.19	-0.88	0.28	4.54	3.47
1995	1.48	1.06	-1.70	0.076	-2.93	-3.14
1996	0.56	1.27	-3.10	0.13	13.49	10.11
1997	0.98	1.39	0.48	0.10	3.40	1.77
1998	1.00	0.73	0.38	0.053	3.27	4.02
1999	1.41	0.57	-21.39	-0.03	35.79	31.97
2000	1.1	2.36	0.00	0.26	1.29	-0.76
2001	1.06	2.33	-0.67	0.082	1.57	2.42
2002	0.92	1.57	-0.26	0.21	4.36	2.78
2003	0.93	2.21	-0.41	0.17	3.95	2.52
2004	1.13	2.37	0.45	0.022	-1.75	1.16
2005	1.79	8.95	1.75	0.48	-4.22	-3.43
2006	1.49	4.67	-0.12	0.15	-4.61	-4.78
2007	0.69	4.46	-1.66	0.09	9.23	4.84
2008	0.61	6.69	-2.80	-0.00	47.51	6.67
2009	0.85	3.69	-1.91	0.35	5.20	1.29

Table 2
Parameter Estimation Annually

m_1	m_2	d	μ	$\overline{X}$	X_0
9.01	4.79	-0.07	0.00	12.90	6.30

Table 3
Parameter Estimation for the Entire Period

2) Demand Function Model) The second specification is the equation for demand function of gasoline given as $P_G = X - bq +_2, \varepsilon_1 = N(0, \sigma_M)$. The problem with this specification is that we do not observe the actual net supply of gasoline to the market at each week. What is reported in the data is the weekly production of refinery sector while the net supply is the sum of refinery production and net changes in gasoline inventories. A complete specification requires modelling of dynamic production-storage problem which is considered in this version of the paper. Table shows estimated values of b in the equation $P_G = m_2 X + m_1 Q + d$ for entire sample (table 3).

A.3. Risk-Neutral Dynamics

We use the futures prices of crude and gasoline in order to estimate market's belief regarding future values of X under risk-risk measure. The observable variables at each time point are two vectors of 12 futures contracts with maturities between 1-12 months for gasoline and crude. This approach allows us to jointly estimate the dynamics of X under both physical and risk-neutral measures and extract what we call the *term-structure of demand process*.

Investigation the shape of crude and gasoline futures suggests (see figure 4 for a sample) that unlike crude, gasoline futures have an strong seasonal element. Therefore, we definte $Q = [Q(T)], T \in \{1, ..., 12\}$ as a vector of montly factors to capture the seasonality effect of each maturity date T.

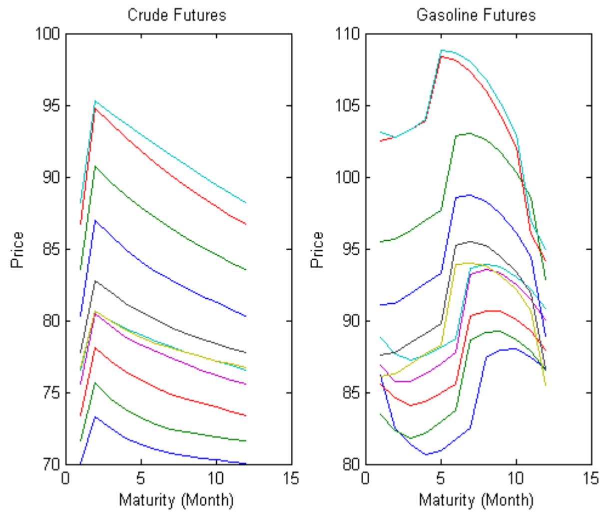

Figure 4. Estimated Values of Demand Process (X), 1990-2010

As before, we assume a single short-term mean-reverting factor for gasoline demand. Following other papers (e.g. Schwartz and Smith (2000) and Manoliu and Tompaidis (2000)) we use the standard formulation as follow. Denote by X(t) the value of stochastic demand parameter at time t. The dynamics of this process under Q-measure is given by $dX = \mu_Q(\overline{X}_Q - X)dt + \sigma_Q dW$. Taking the expectation and multiplying by the relevant seasonality factor gives the expected value of X at any future maturity time T. $E(X(t,T)) = Q(T)(X(t)e^{-\mu_Q(T-t)} + \overline{X}_Q(1-e^{-\mu_Q(T-t)}))$.

If we rely on linear structure for the relationship between gasoline and crude prices, then $E^Q_{t,T}(P_G) = m_1 E^Q_{t,T}(P_C) + m_2 E^Q_{t,T}(X)$. For any given set of initial values and parameters $\{Q(T), X(t), \mu_Q, \overline{X}_Q\}$ and observed crude prices (F_{CL}), expected gasoline price $E^Q_{t,T}(P_G)$ and the estimation error $e_{t,T} = F_{Gas}(t,T) - E^Q_{t,T}(P_G)$ be calculated. Kalman filter will choose the parameters of P and Q dynamics in such a way to maximize a likelihood function which depends on all $e_{t,T}$.

m₁	m₂	μ_Q	$\overline{X}_Q$	μ_X	$\overline{X}$	X₁
1.0378	3.3972	-0.0000	58.3293	0.0535	3.0249	1.6231

Table 4
Joint Physical and Risk-neutral Parameter Estimation

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1.00	0.87	1.20	1.07	0.90	0.79	0.81	0.89	1.13	0.87	0.86	0.88

Table 5
Monthly Factors

To summarize, the state-space representation of problem would be as follow:

$Y_t = M_1 d_t + M_1 X + M_2 Z,$

$Y_t = [F_{Gas}(t,T)]$ and $Z = [F_{Crude}(t,T)]$

$d_t = [Q(T)(\overline{X}_Q(1 - e^{-\mu_Q(T-t)}))],$

$X = [Q(T) X_t e^{-\mu_Q(T-t)}]$

$X_{t+1} = X_t e^{-\mu} + (1 - e^{-\mu})\overline{X}$

$T \in \{1, ..., 12\}$

The estimation results are summarized in tables 4 and 5

$\mu_Q = 0$ and $Q(T) > 0, \forall T \in \{1, .., 12\}$ suggest that all the expectation regarding the future values of X under risk-neutral measure is captured by the seasonality factors. More formally, $\mu_Q = 0 \Rightarrow dX = \sigma_Q dZ \Rightarrow E(X(t,T)) = Q(T)X(t)$d.

IV. Conclusions

We have presented an equilibrium model for the price of the output in a supply chain, given a stochastic process for the supply of the input, the demand of the output and a capital asset that transforms the input into the output and faces operational constraints. We calibrated our model to the case of a refinery turning crude oil into gasoline. One application of the model is to determine the optimal hedging policy a refinery. For example, a consequence of our model is that, if demand for gasoline is weak, input and output prices follow almost the same path with a fixed crack spread. In this case the refinery does not need to hedge, since changes in the price of the input are offset by changes in the price of the output, leaving a fixed margin for the refinery. On the other hand, if demand for gasoline is strong, output and input prices are no longer highly correlated, resulting in a more volatile crack spread. Using the dynamics of the crack spread from our model, the refinery can decide whether a hedge of input or output is necessary and then calculate the optimal hedge policy.

Another application of our model is to use the estimated cashflows under the risk-neutral measure to estimate the value of the financial contracts such as futures or options on gasoline, or even the value of the refinery itself. Beyond simply valuing the refinery we can also derive the dynamics of the covariance of the value of the refinery with the market portfolio — beta — given information on the covariance of the stochastic processes of the supply of crude oil and the demand for gasoline with the market portfolio. We leave these applications for future research.

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