Control of Accumulating Stock Pollution by Heterogeneous Producers *

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Abstract

Activities by heterogeneous producers are the cause of a wide range of environmental problems where environmental quality depends on an aggregate pollution stock that accumulates over time. The stock of pollution can be reduced by changing production practices or by removing pollutants after they are generated. In this paper we analyze the optimal combination of source control and abatement strategies over time. The source control is given by the input use and the technology choice for each unit. The results show that that the optimal intertemporal policy depends on the form of abatement cost and damage functions and that input taxes need to be complemented by technology taxes or subsidies to establish the socially optimal outcome. Finally, the paper presents a novel approach for solving spatial-intertemporal optimal control problem - the so called two stage solution approach.

Key words: input use, technology adoption, abatement, nonpoint-source pollution, optimal control, technology policies, input taxes

JEL Classification: Q18, Q24 and H23

1 Introduction

Over the last decades, it has been a growing concern about pollution problems including surface and ground water contamination, climate change, animal waste among many others, where pollution is generated by a large number of units, it is aggregated and accumulates over time. For example, when producers use fertilizers to increase productivity, some of the applied fertilizer deep percolates and contaminates ground water, but the damage increases over time as the concentration of chemical in the water increases. However, the private incentives to decrease pollution might not be sufficient since producers do not take into account the externalities resulting from their activities, requiring government intervention.

Policy makers can address the problem in two ways, they can enact policies that induce changes in production practices (control at the source) or they can remove the pollutants after they have been generated but before they cause environmental damage (abatement at the receptor). Source control can be reached by a reduction of input use, that is affecting the intensive margin of production, by the exit of some producing units (extensive margin), or by managing the use of inputs more efficiently, encouraging the adoption of conservation technologies (Khanna and Zilberman, 1997; Fuglie and Kascak, 2001). On the other hand, there are specific measures to address the pollution after it has been generated to reduce its environmental effects, e.g., retention ponds, solids separation basins, wetland buffers and vegetative practices such as filter strips between production facilities and nearby surface waters (Mitsch et al., 1999).

This paper develops a modelling framework to address pollution problems where environmental quality depends on an accumulating pollution stock and producers are heterogeneous. It characterizes the optimal resource allocation and identifies policy instruments that will lead to first best outcomes for competitive industries under different informational assumptions. The model is most appropriate for agricultural situations where the heterogeneity is given by the land quality but it can be applied to other situations, for example when there is a "putty clay" technology with assets of various inputs as in the case of the energy sector or when capital goods have long economic life and production is done in plants of different characteristics or using various machines.

The paper investigates the dynamics of key variables that determine the optimal incentive to conservation versus abatement over time. The control at the source is determined by the allocation of the different technologies, the number of producing units (extensive margin), and the intensity of production, that is the optimal level of input use (intensive margin). The abatement is taken into account by considering the possibility of treating the pollutant after it has been generated in order to reduce the concentration of the pollutant at the receptor. Moreover, it is determined the optimal combination of policies affecting the intensive margin (e.g. input taxes) and the extensive margin (technology taxes or subsidies) in order to induce the socially optimal technology choice.

The modelling approach taken here is a full-information control strategy where outputs are observable and there is no production uncertainty. The results show that the intertemporal optimal policy depends on the convexity of abatement cost and damage functions. When the accumulation of pollution stock produces a significant increment on the the cost of abatement and on the environmental damage or when abatement cost is highly convex with respect to the level of abatement, i.e., marginal abatement cost increases rapidly with the abated quantity, the abatement effort at the receptor is not sufficient to reduce the stock of pollution to its optimal intertemporal path and it has to be reinforced with a high control at the source, implying a decrease in the aggregate emissions and consequently a decrease in the resource use. In contrast, if the marginal abatement cost increases slowly with additional abatement, the optimal intertemporal policy is characterized by a high abatement at the receptor. In this case, the decrease in aggregate emissions and in the input use towards the steady state values is done gradually. The results also demonstrate that a policy based on the reduction of input use alone is not able to achieve the social optimum since it produces a distortion on the extensive margin and therefore it must be complemented with technology policies that encourage the optimal allocation of the conservation technologies.

A very essential part of this paper consists of the novel approach in presenting a two-stage solution to a spatial-intertemporal optimal control problem. The two-stage solution, allows us to derive the qualitative characteristics of the solution better and more easily than a single-stage solution. The first stage consists of the optimal static solution taking into account heterogeneity conditions, and the second stage comprises the intertemporal optimization of the solution of the first stage. The two stages are linked by the common shadow price that allows the necessary changes needed to transform a spatially optimal, yet static, environmental policy analysis to an intertemporally and spatially optimal environmental policy.

The organization of the paper is as follows. Section 2 reviews the literature. Section 3 describes the basic features of the model and in section 4 we find the optimal environmental policy, divided into the optimal static solution and the optimal intertemporal solution. Section 5 analyze the spatial and intertemporal input and technology policies that can achieve the social optimum. Finally section 6 concludes the paper.

2 Review of the literature

There is a wide array of cases in what pollution by heterogeneous producers is aggregated and accumulates over time. *Some examples are summarized in table 1.* Most of the recent literature is related to the adoption of a conservation technology as a part of solution to the stock externality problem. Examples on the agricultural sector include the use of modern irrigation technologies such as sprinkles, drip and other volume irrigation to reduce the amount of water applied and hence the leaching of pollutants (Caswell and Zilberman, 1985; Caswell et al., 1990; Green et al., 1996), the adoption of soil Nitrogen testing to adjust more precisely the amount and timing of input application to crop growth needs in order to reduce potential N-losses (Fuglie and Bosch, 1995), the adoption of Integrated Pest Management to decrease pesticide use (Moffitt, 1993; Abler and Shortle, 1995; Fernandez-Cornejo, 1998), and the adoption of conservation tillage that can reduce soil erosion and water and air pollution (Hu et al., 1997; Pautsch et al., 2001). Moreover, Lal et al., (1998) have shown that the use of minimum tillage can increase carbon sequestration rates, decreasing green house gas emissions so that it contributes to moderate the global warming. Similar benefits of precision technologies have been investigated in different production processes, e.g. in the transportation sector (Khazzoom, 1995; Michaelis, 1995; Nakata, 2000), in the electricity-generating sector (Siegel and Temchin, 1991; Khanna and Zilberman, 1999; Barnali and Parikh, 2000??), and in the manufacture of iron, steel, cement and glass among many others (Tester et al., 1991). Most of these studies support the finding that taking into account the heterogeneity of producing units is a crucial issue in the study of technology adoption.

There is a second group of papers focusing on the regulations in the extensive margin to reduce pollution stock problems. Ribaudo et al. (1994) study crop land retirement as an option for reducing water pollution. Their results show that land retirement as a primary pollution control tool is expensive, but if it is appropriately targeted, could generate sufficient benefits to compensate the costs. Wu and Segerson (1995) present an empirical framework for quantifying effects of commodity programs and taxes on the extensive margin and consequently on potential groundwater pollution in Wisconsin and Plantinga (1996) examine the potential gains in environmental quality resulting from price incentives to change the extensive-margin.

A third group of models is concerned on the optimal combination of source control and abatement policies. Shah et al. (1995) present a dynamic framework to analyze the optimal combination of on-farm and off-farm pollution control measures in the problem of water logging, however assuming constant land quality. Ribaudo et al. (2001) use an empirical model to evaluate the impacts of alternative strategies to achieve a particular Nitrogen reduction in the Mississippi Basin. They find that reducing fertilizer use is less costly than wetland restoration up to a level of nitrogen loss reduction, beyond this point, wetland restorations are most cost-effective.

Farzin (1996) develops a dynamic framework to analyze how should the static policy instruments be modified in the presence of an stock externality problem and simulates the model for the case of fossil fuel burning and the consequent global warming.

However, all these studies focus on specific parts of the pollution abatement process. They analyze either the optimal intertemporal policies assuming homogeneity in production units or they optimize with respect to quality without incorporating the intertemporal aspect of the pollution stock problem. In this paper we incorporate heterogeneity as well as intertemporal aspects together in the pollution abatement decision process. The consideration of heterogeneity allows us to determine the effects of a change in the quality of the fixed asset in the extensive and intensive margins and on the conservation technology adoption rates, whereas the dynamic framework together with allowing the possibility of on-farm and off-farm measures to decrease the pollution stock, makes it possible to evaluate the incentives to control the pollution at the source versus abating it after it has been generated. Moreover, it also allows us to evaluate the effects of the policies over the resource use in the long-run.

3 The Economic Model

Consider a production process that uses a variable input and a fixed input (an asset) to produce a single output. The producing units (for example, farms, small firms or households) differ in the quantity and quality of the fixed asset they own. The heterogeneity of the asset is denoted by ϵ, $\epsilon \in [\epsilon_0, \epsilon_1]$, reflecting the quality of the production unit. It may represent a measure of land quality, but it may be vintage in case of machinery or even differences in management practices. It is assumed that the higher is the value of ϵ the higher is the asset quality. The available asset with quality ϵ is denoted by $X(\epsilon)$.

For simplicity, we concentrate on the case where two alternative technologies i, $i = 1, 2$ are available. The subscript i=1 represents a precision technology, and i=2 denotes the traditional technology. The traditional and precision technologies may be types of fertilizer, different irrigation technologies or pesticide management practices in agricultural activities or two different coal qualities in the case of electricity generation. The asset allocated to technology one at any moment of calendar time t, with quality ϵ is denoted by $x_1(t,\epsilon)$ and the asset allocated to technology two is given by $x_2(t,\epsilon)$. Let $u_1(t,\epsilon)$ and $u_2(t,\epsilon)$ represent the applied input per unit of asset associated to technology 1 and 2 respectively and let $e_i(t,\epsilon)$ be the effective input per unit of asset using technology i. The ratio of effective input to applied input is called effectiveness of input use and it is denoted by β_i, it depends on the technology used, where $\beta_1 > \beta_2$, that is, the conservation technology has a higher effectiveness of input use.[1].

Let $f(e_i(t,\epsilon), x_i(t,\epsilon))$ represent the C^2 production function under technology i. We assume constant returns to scale with respect to the fixed input, thus we can rewrite the function as $f(e_i(t,\epsilon))x_i(t,\epsilon)$, and it strictly concave in e_i, that is, $f_{e_i} > 0$ and $f_{e_ie_i} < 0$, where the subscript of a function with respect to a variable denotes its partial derivative. The assumption of constant returns to scale within a production unit has been widely applied in different models, for example in a power plant unit (Khanna and Zilberman, 1999) or in a firm (Caswell et al., 1993). The product and input prices are denoted by p and c respectively. We assume that annualized fixed costs of technology adoption per unit of asset I, are larger for technology 1 than those required for technology 2, i.e. $I_1 > I_2$.

The heterogeneity also affects the productivity of the fixed asset under each technology. Thus, $y_i = h_i(\epsilon)f(e_i(t,\epsilon))$, $i = 1, 2$, reflects the production per unit of asset with quality ϵ using technology i, where $h_i(\epsilon)$ represents the productivity of the fixed asset. It may represent variability in soil fertility, or the productivity of the different machineries. We assume that $dh_i/d\epsilon > 0$, i.e., the productivity increases with the asset quality.

[1] For example, in the case of irrigation technologies, precision technologies such as sprinkler or drip irrigation can reach an irrigation effectiveness of 0.95 while efficiency of traditional irrigation methods is only about 0.6 (Hanemann et al., 1987).

The applied input that is not utilized by the production process can be a source of environmental contamination. We define the pollution function per unit of asset of quality ϵ associated with each particular technology choice i, $i = 1, 2$ as $\alpha_i(\epsilon)g(u_i(t, \epsilon))$ where $g(u_i(t, \epsilon))$ represents the potential emissions per unit of asset as a function of the input applied with technology i. We assume that $g_{u_i} > 0$, $g_{u_i u_i} > 0$, i.e. the marginal pollution is increasing in the input use, and $\alpha_i(\epsilon)$ indicates the part of the potential emissions that reach the receptor for a given quality. We assume that $\alpha_1(\epsilon_1) = \alpha_2(\epsilon_1) = 0$ and $d\alpha_2/d\epsilon < d\alpha_1/d\epsilon < 0$. The former property indicates that the emissions that reach the receptor are zero when the quality is maximum. The latter property together with the former implies that the functions cannot intersect, thus technology 2 is more polluting than technology 1, and also indicates that $\mid \alpha_1(\epsilon) - \alpha_2(\epsilon) \mid$ increases as ϵ decreases, in other words, technology two exacerbates the potential emissions as the asset quality decreases. The emissions of activity one and two with quality ϵ that accumulate at the receptor are given by $\alpha_1(\epsilon)g(u_1(t, \epsilon))x_1(t, \epsilon)$ and $\alpha_2(\epsilon)g(u_2(t, \epsilon))x_2(t, \epsilon)$ respectively.

The economic loss resulting from the emissions is given by the damage function $d(s(t))$, where $s(t)$ is the concentration (i.e., stock) of the pollutant in the receptor at time t. We propose a C^2 damage function with the following properties: $d(0) = 0$, $d_s > 0$, $d_{ss} > 0$, and $d(s) = N$, $\forall s \geq \bar{s}$, where $0 < N < \infty$. One can think of $\bar{s}$ as the saturation point of the concentration of the pollutant in the receptor

To reduce the concentration of the pollutant, abatement strategies are available at the regional level, in form of treatments at the receptor. Thus, let $\eta(t)$ denote abatement at period t, and let the abatement cost function be denoted by $k(\eta(t), s(t))$. It is reasonable to assume that the marginal cost of abatement is positive $k_\eta > 0$ and increasing with the abated quantity $k_{\eta\eta} > 0$, but decreasing with the stock of pollution $k_{\eta s} < 0$. We also assume that $k_s > 0$ and $k_{ss} > 0$, since the growing social awareness could generate the need to perform more intensive treatments of the pollution stock, increasing the abatement costs. *Figure 1* presents a possible scheme of the key variables and their relationships in the production process.

Figure 1

The dynamics of the concentration of the pollutant in the receptor can now be stated as

$$\dot{s}(t) = \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(t,\epsilon)) x_i(t,\epsilon) \Big) \, d\epsilon - \eta(t) - \zeta s(t), \qquad (1)$$

where a dot over a variable denotes the operator $\frac{d}{dt}$. Following Clasen et al. (1989), we describe the concentration of the pollutant over time as a linear function in s. The parameter ζ, $0 < \zeta < 1$ represents the natural decay rate of the pollutant at the receptor.

4 Dynamics of the pollution stock problem

It is assumed that a social planner exists and maximizes the present discounted value of the net benefits from production while taking into account the social economic losses due to the accumulation of the pollutant. [2] Thus, the social planner's decision problem is given by

[2]Given the regional focus of the analysis, we assume that the product prices are not influenced by regional production decisions and thus they are taken as given. Thus, the output price is also not influenced by the production of the externality. We assume that there are no transportation costs, and that the utility function of the consumers is quasilinear with respect to the traded goods and the externality. Thus, the optimal level of the externality is independent of the consumers' expenditures, and it is possible to derive a utility function which depends only on the externality s (Mas-Colell, Whinston and Green, 1995). To discuss the results of our model in a practical setting we propose that the derived utility function can be represented by the damage function $d(s(t))$ and the abatement cost function $k(\eta(t), s(t))$. Additionally it is also assumed that there are no cost of public funds, and lump sum transfers are available to redistribute income so that land-use taxes are not distortionary (Sandmo, 1995). The assumptions made with respect to the quasilinearity of the utility function and the existence of costless public funds help to keep the model simple. It allows us to concentrate our analysis on the incentives to control pollution versus abating it and on the design of environmental policies.

$$\max_{u_i(t,\epsilon),\, x_i(t,\epsilon),\, \eta(t)} \quad \int_0^\infty \exp^{-\delta t}\Big[\int_{\epsilon_0}^{\epsilon_1}\Big(\sum_{i=1}^{2}\big(ph_i(\epsilon)f(\beta_i u_i(t,\epsilon)) - \\ -cu_i(t,\epsilon) - I_i\big)x_i(t,\epsilon)\Big)d\epsilon - \Big(d(s(t)) + k(\eta(t), s(t))\Big)\Big]\, dt, \qquad (S)$$

subject to

$$\begin{aligned}
\dot{s}(t) &= \int_{\epsilon_0}^{\epsilon_1}\Big(\sum_{i=1}^{2}\alpha_i(\epsilon)g(u_i(t,\epsilon))x_i(t,\epsilon)\Big)\, d\epsilon - \eta(t) - \zeta s(t), \\
& s(0) = s_0, \quad u_i(t,\epsilon) \geq 0,\ i = 1,2, \quad x_i(t,\epsilon) \geq 0,\ i = 1,2, \\
& x_1(t,\epsilon) + x_2(t,\epsilon) \leq X(\epsilon) \quad 0 \leq \eta(t) \leq s(t).
\end{aligned}$$

where s_0 denotes the amount of pollution stock at the receptor at the initial point of calendar time, and the parameter $\delta > 0$ denotes the social discount rate.

Proposition 1 *The dynamic optimization problem S is equivalent to specifying and solving two sequential problems $S1$ and $S2$. Thus, the solution of the general problem S can be determined in two stages. In the first stage the model is maximized over ϵ subject to a prespecified level of aggregate emissions z, obtaining the optimal trajectories of $u(\epsilon)$ and $x(\epsilon)$ (problem $S1$). In the second stage the parameter z becomes a decision variable and the optimal trajectories of the functions $u(t,\epsilon)$, $x(t,\epsilon)$ and $\eta(t)$ over time are obtained (problem $S2$).*

The proof is in the appendix. Because of the structure of the problem, we are able to decompose it into a static control problem and a dynamic control problem. The static control problem optimizes the use of resources over quality (or space) determining how much each unit should control pollution to maximize temporal benefits given aggregate pollution constraints. The solutions of this problem are plugged in the dynamic control problem to assess the optimal combination of pollution control at the source given by the aggregate emissions and abatement at the receptor.

Separating the problem in two stages is possible given the fact that the state variable is an aggregate function that only depends on time. In the first stage we analyze the optimal solution over ϵ, i.e., the optimal level of applied input, the optimal technology choice for each unit, and the optimal riparian zone, given by $X(\epsilon) - \big(x_1(\epsilon) + x_2(\epsilon)\big)$, at every location ϵ, that is the optimal range of qualities in what production does not take place. In the second stage we derive the optimal intertemporal solution for the optimal static solution given by the optimal combination of pollution control at the source and abatement at the receptor.

4.1 The Optimal Static Solution

In the first stage the solution of the social planner's decision problem is given by the value function $V(z)$ defined as:

$$V(z) \equiv \max_{u_i(\epsilon),\, x_i(\epsilon)} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(ph_i(\epsilon) f(\beta_i u_i(\epsilon)) - cu_i(\epsilon) - I_i\big) x_i(\epsilon) \Big)\, d\epsilon \qquad (S1)$$

subject to

$$z = \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(\epsilon)) x_i(\epsilon) \Big)\, d\epsilon,$$

$$u_i(\epsilon) \geq 0,\ i = 1, 2, \quad x_i(\epsilon) \geq 0,\ i = 1, 2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon),$$

where z denotes the aggregate emissions over the entire range of ϵ, from ϵ_0 to ϵ_1, which accumulates over time.

A solution of the problem has to satisfy the following necessary conditions

$$\mathcal{L}1_{u_i} \equiv (ph_i \beta_i f_{u_i} - c - \lambda \alpha_i g_{u_i}) x_i + \upsilon_i = 0, \qquad (2)$$

$$\mathcal{L}1_{x_i} \equiv py_i - cu_i - I_i - \lambda \alpha_i g(u_i) + \upsilon_{i+2} - \upsilon_5 = 0, \qquad (3)$$

$$\mathcal{L}1_{\lambda} \equiv z - \int_{\epsilon_0}^{\epsilon_1} \big(\sum_{i=1}^{2} \alpha_i g(u_i) x_i \big)\, d\epsilon = 0. \qquad (4)$$

The Lagrange multiplier λ is interpreted as the shadow costs of the prespecified level of emissions at the receptor, z. Please note that z does not depend on ϵ, thus λ is constant over space. Given the interpretation of λ, the necessary condition (2) indicates that the value of the marginal product of applied input per unit of asset of quality ϵ with each technology should equal the sum of its marginal private cost and the marginal cost of pollution per unit of asset for an interior solution. In the case of a boundary solution, the Lagrange multiplier of the binding constraint reflects the difference between the value of the marginal product and the sum of the marginal costs. Since the production function has constant returns to scale, we will find an optimal level of input use per unit of asset with quality ϵ ($\forall x_i > 0$) irrespective of the total asset endowment.

Proposition 2 *For a given technology, an increase in the quality of the asset leads to an increase in the input use and to an increase in the output.*

$$\frac{\partial u}{\partial \epsilon} > 0, \qquad \frac{\partial y}{\partial \epsilon} > 0.$$

The proof is shown in the appendix. Equation (A. 21) implies that an increase in the asset quality increases the optimal intensity of input use of either technology since it increases the productivity of the marginal unit of the applied input and it decreases the marginal pollution level. In the case that β_i depends on ϵ as in Caswell and Zilberman (1986), the change on input use would depend on the elasticity of marginal productivity.

The necessary condition (3) indicates that the marginal net benefits of production per unit of asset of quality ϵ with each technology should equal the marginal cost of pollution per unit of asset. However, since both production an emissions functions are linear in the fixed asset, the technology that leads to a higher quasirent, say $\Pi_i^* \equiv \mathcal{H}1_{x_i}$ will be completely preferred to the technology with the lower quasirent, implying that all the resources with quality ϵ should be used with the technology that yields the highest quasirent. Hence, we exclusively obtain boundary solutions for every quality ϵ given either by $x_1(\epsilon) = X(\epsilon)$, $x_2(\epsilon) = X(\epsilon)$ or $x_1(\epsilon) = x_2(\epsilon) = 0$. In this case, the Lagrange multiplier of the binding constraint reflects the difference between the value of the marginal net benefits and the marginal pollution cost. The adoption of the conservation technology is optimal when the quasirent of the precision technology is positive and larger than that of the traditional technology. The difference in quasirent per unit of asset of quality ϵ with the modern and traditional technology can be written as:

$$\Pi_1^* - \Pi_2^* \equiv \mathcal{H}1_{x_1} - \mathcal{H}1_{x_2} = p\triangle y^* - c\triangle u^* - \triangle I - \lambda(\alpha_1 g(u_1^*) - \alpha_2 g(u_2^*)) \quad (5)$$

where $\triangle$ represents the difference in the level of the different variables with the two technologies. Equation (5) implies that the precision technology should be adopted if the impact of adoption in the increase in output, input saving and decrease in pollution compensates the difference in the fixed cost required to implement the technology, that is, the higher annual expenditures per unit of asset on human capital and equipment. However, the maximal quasirent for technology i, Π_i^* depends on the asset quality, and thus it will change over ϵ. As equation (A. 22) shows, the quasirent for each technology increases with an increase in the quality ϵ since it increases the asset productivity and it decreases the amount of emissions that reach the receptor. Furthermore, we can see from (A. 23) that the quasirent is more likely to be convex in the low qualities because when the input use is low, the production and pollution are low while the derivatives f_{u_i} and g_{u_i} tend to infinity. As the asset quality increases, the first term in brackets increases while the second decreases, and therefore the quasirent is more likely to be concave.

In order to give more insight to the analysis let us assume that $\beta_1 = \beta_2 = 1$. Although this assumption doesn't change the dynamic aspect of the model, it will be useful to derive some characteristics of the adoption pattern. At the maximum quality ϵ_1, the traditional technology will be preferred to the modern technology since it has lower costs of adoption ($\Pi_1^*(\epsilon_0) < \Pi_2^*(\epsilon_0)$). As the asset quality declines, the modern technology becomes more profitable as activity 2 exacerbates the potential emissions. In the case that the order of the inequality is reversed, we know that the social quasirent functions intersect and it is optimal to diversify the technology choice. The switching point is given by ϵ^* where $\Pi_1^*(\epsilon^*) = \Pi_2^*(\epsilon^*)$. This case is depicted in *Figure 2* for the quasirents Π_1^* and Π_2^*. Thus traditional technologies are optimal for high levels of ϵ where the potential emissions are low. According to an economic criterion, the traditional technology is preferred because it has lower annualized fixed costs of adoption per unit of asset. On the other hand precision technologies are optimal for $\forall\epsilon < \epsilon^*$. In this case the environmental criterion prevails over the economic criterion, that is, the higher productivity together with the lower emissions of the modern technology with respect to traditional technology overcomes the costs of adopting the precision technology.

Figure 2

Additionally, *Figure 2* also presents the case where both quasirents $\Pi_1^{*\prime}$ and $\Pi_2^{*\prime}$ turn negative below the quality $\bar{\epsilon}$, $\bar{\epsilon} \in [\epsilon_0, \epsilon_1]$. Hence, no production will take place on range of qualities between ϵ_0 and $\bar{\epsilon}$ [3].

If the order of the inequalities of (a) and (b) is not reversed, the quasirents of activity one and two do not intersect and therefore, it is optimal not to diversify the technology choice. The traditional technology will be preferred because of the lowest costs of adoption. *Figure 3* illustrates this case. Likewise as in *Figure 2*, there may exist a $\bar{\epsilon}$ below which no production takes place, as given in *Figure 3*.

Figure 3

Now we will relax our initial assumption of $\beta_1 = \beta_2 = 1$ to see how the difference in productivity of the input use can affect the adoption pattern. In this case, at the highest quality, the modern technology will conserve resources, and it will be preferred than the traditional when the decrease in the input use overcomes the higher costs of adoption, that is when

$$(\frac{\beta_2}{\beta_1} - 1)cu_2^* > I_1 - I_2 \tag{6}$$

This case is depicted in *figure 4*.

Figure 4

[3]Even though $X(\epsilon)$ presents the total asset of quality ϵ, and clearly it is variable over ϵ, we simplify the graphical presentation of our results by setting $X(\epsilon)$ constant over ϵ.

4.2 The Optimal Dynamic Solution

In the first stage we derived the socially optimal spatial solution, given by the optimal allocation of the technologies and the optimal level of input use from a static point of view. To analyze how the optimal solution is affected over time, we maximize the value function V obtained in the first stage over time. Hence, the social planner's decision problem is given by:

$$\max_{z(t),\,\eta(t)} \int_0^\infty exp^{-\delta t}\Big(V(z(t)) - d(s(t)) - k(\eta(t), s(t))\Big)\, dt \tag{S2}$$

subject to

$$\begin{aligned} \dot{s}(t) &= z(t) - \eta(t) - \zeta\, s(t), \\ & \quad s(0) = s_0, \quad \eta(t) \leq s(t), \quad z(t) \in \mathcal{Z} = [0, \bar{z}]. \end{aligned}$$

The parameter z introduced in the first stage problem, that denotes the emissions of the entire region that reach the receptor, becomes the decision variable in the second stage. Thus, it now depends on t. The upper limit of the set $\mathcal{Z}$, $\bar{z}$, corresponds to the highest possible emission evaluated at the receptor. Thus, the decision variables in the intertemporal allocation are given by the emissions received by the receptor $z(t)$ (i.e., indirectly, the abatement effort at the source level) and the abatement effort at the receptor $\eta(t)$ so that we will be able to analyze the optimal mix of reducing emissions at the source versus abatement at the receptor. The first order conditions for an interior solution read as

$$V_z = k_\eta = \varphi \tag{7}$$

$$\dot{\varphi} = \delta\varphi + \mathcal{H}2_s = \big(\delta + \zeta\big)\varphi - d_s - k_s, \tag{8}$$

$$\dot{s} = z - \eta - \zeta s, \qquad s(0) = s_0. \tag{9}$$

Equation (7) states that the marginal value of the aggregate emissions of the producing units evaluated at the receptor should equal the marginal cost of abatement which at the same time should equal the marginal shadow cost φ. Equation (8) indicates the change in the shadow cost of a delayed reduction of a marginal unit of the pollution stock from period t to period $t+1$.

For a sustainable abatement policy the social planner is particularly interested in the achievement of a steady state, defined by equations (8) and (9) as $\dot{\varphi} = \dot{s} = 0$. Assuming an interior solution, equation (7) can be solved globally and uniquely by using Theorem 6 in Gale and Nikaidô (1965) for $z = \hat{z}(\varphi, s)$ and $\eta = \hat{\eta}(\varphi, s)$ (for details, see the Appendix). For the purposes of a qualitative analysis we reduce the necessary conditions (7) - (9) to a pair of differential equations in φ and s by substituting $z = \hat{z}_(\varphi, s)$ and $\eta = \hat{\eta}(\varphi, s)$ into (8) and (9) to get

$$\dot{\varphi} = \big(\delta + \zeta\big)\varphi - d_s - k_s(\hat{\eta}(\varphi, s), s), \tag{8 $'$}$$

$$\dot{s} = \hat{z}(\varphi, s) - \hat{\eta}(\varphi, s) - \zeta s, \qquad s(0) = s_0. \tag{9 $'$}$$

A linearization of the canonical system of differential equations around the steady-state values of φ and s results in

$$\begin{pmatrix} \dot{\varphi} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} \geq 0 & \frac{\partial \dot{\varphi}}{\partial s} \gtreqless 0 \\ \frac{\partial \dot{s}}{\partial \varphi} \leq 0 & \frac{\partial \dot{s}}{\partial s} \leq 0 \end{pmatrix} \begin{pmatrix} \varphi - \varphi^{\infty} \\ s - s^{\infty} \end{pmatrix}. \tag{10}$$

Since the trace of the Jacobian matrix, trJ is equal to $\delta > 0$, employing the fact that the trJ equals the sum of its eigenvalues assures that at least one eigenvalue is positive. First, we have considered the case that $\frac{\partial \dot{\varphi}}{\partial s} < 0$. In this case, the determinant of the Jacobian matrix is negative and thus, the eigenvalues have opposite signs and the steady state equilibrium is locally characterized by a saddle point. However, in the case that $\frac{\partial \dot{\varphi}}{\partial s} > 0$, the determinant of the Jacobian matrix can be negative or positive. If it is negative the steady state equilibrium is also locally characterized by a saddle point, however, in the case that the expression $\frac{\partial \dot{\varphi}}{\partial s} > 0$ leads to a positive determinant of $\tilde{J}$, the steady state is an unstable equilibrium. Nevertheless, this case is economically not relevant, thus, our analysis concentrate on the first two cases.

For any initial value of s within the neighborhood of s^{∞}, where the superscript ∞ indicates the steady state equilibrium value, it is possible to find a corresponding value of the shadow cost which assures that the optimal environmental abatement policy leads towards the long-run optimum.

As it is stated in the expression of $\partial\dot{\varphi}/\partial s$ in equation (A. 26) an increase in a marginal unit of the pollution stock produces an increase in the marginal environmental damage and additionally, it affects the abatement cost in two opposite ways, it has a direct increasing effect in the abatement cost function caused by the more intensive treatments at the receptor ($k_{ss} > 0$) and an indirect effect in the form of a smaller marginal cost of abatement given by the last term, as the increase in the pollution stock reduces the price of abating a specific quantity of pollution. Thus, the global effect in the evolution of the shadow cost depends on the relative magnitude of those opposed effects. The following proposition establishes the optimal path approaching to the steady state as a function of the relative slopes of the abatement and damage cost functions.

Proposition 3 *: The stable path leading to the steady state can be upward or downward sloping depending on the convexity of damage and abatement cost functions. When the increase in the marginal damage and abatement cost overcomes the indirect effect, i.e.* $d_{ss} + k_{ss} > |k_{\eta s}\frac{\partial \eta}{\partial s}|$*, the shadow cost will increase with an increase in the stock of pollution, decreasing otherwise.*

This proposition is illustrated in *figures 5* and *6* below, however, readers interested in a more rigorous proof can obtained it from the authors. When $d_{ss} + k_{ss} > |k_{\eta s}\frac{\partial \eta}{\partial s}|$, the slopes of the $\dot{\varphi} = 0$ and $\dot{s} = 0$ isoclines of the phase diagram in the (s, φ) space are

$$\left.\frac{d\varphi}{ds}\right|_{\dot{\varphi}=0} = -\frac{\frac{\partial \dot{\varphi}}{\partial s}}{\frac{\partial \dot{\varphi}}{\partial \varphi}} > 0, \qquad \left.\frac{d\varphi}{ds}\right|_{\dot{s}=0} = -\frac{\frac{\partial \dot{s}}{\partial s}}{\frac{\partial \dot{s}}{\partial \varphi}} < 0. \tag{11}$$

The resulting phase diagram depicted in *Figure 5* shows that the stable path leading to the steady state is upward sloping while the unstable path is downward sloping. In this case the pollution stock and its shadow cost evolve in a parallel way, and therefore any pollution abatement policy can be depicted by a decrease in the shadow cost.

Figure 5

On the other hand, when $d_{ss} + k_{ss} < |k_{\eta s}\frac{\partial \eta}{\partial s}|$, either because the increase in the marginal damage and abatement costs associated with an increase in the pollution stock are low or because the indirect effect is high enough to overcome the direct effects, the shadow cost will decrease with an increase in the aggregate pollution stock.

In this case the slopes of the $\dot{\varphi} = 0$ and $\dot{s} = 0$ isoclines are both negatives and it holds that

$$\left.\frac{d\varphi}{ds}\right|_{\dot{\varphi}=0} = -\frac{\frac{\partial \dot{\varphi}}{\partial s}}{\frac{\partial \dot{\varphi}}{\partial \varphi}} \qquad > \qquad \left.\frac{d\varphi}{ds}\right|_{\dot{s}=0} = -\frac{\frac{\partial \dot{s}}{\partial s}}{\frac{\partial \dot{s}}{\partial \varphi}}. \tag{12}$$

The resulting phase diagram is depicted in *Figure 6.* It shows that the stable path leading to the steady state is upward sloping. Thus, an increase in the stock of pollution will reduce the shadow cost of emissions. Assuming that $s_0 > s^{\infty}$, the concentration of the pollutant is decreasing along the optimal path, while the shadow cost is increasing. Therefore, as *Figure 6* shows, any pollution abatement policy can be depicted by an increase in the shadow cost φ.

Figure 6

In the particular cases where there is no possibility of abatement at the receptor ($\eta(t) = 0, \ \forall t$) or the abatement cost does not depend on the pollution stock ($k = k(\eta)$), the phase diagram is exclusively given by *Figure 5.*

As the convexity of the environmental damage and abatement cost functions determine the optimal intertemporal path, they also establish the optimal combination of source control and abatement policies over time. Moreover, it also allows us to determine the evolution of the optimal input demand function over time. Thus, we can derive the optimal relationship between short-run and long-run input demand functions.

Proposition 4 *: Given an initial stock of pollution greater than s^{∞}.*

1. *If* $d_{ss} + k_{ss} > |k_{\eta s}\frac{\partial\eta}{\partial s}|$*, the optimal dynamic policy consists on the initial choice of aggregate emissions* z'*, and consequently on the initial choice of the input use* u_i'*,* $i = 1, 2$*, below their steady state values* z^{∞} *and* u_i^{∞} *and their gradual increase until the steady state values are reached, and on the initial choice of abatement at the receptor* η' *above its steady state value* η^{∞} *and its gradual decrease until the steady state value is reached.*

2. *If* $d_{ss}+k_{ss} < |k_{\eta s}\frac{\partial\eta}{\partial s}|$*, the optimal dynamic spatially differentiated policy consists on the initial choice of aggregate emissions* z'*, and consequently on the initial choice of the input use* u_i'*,* $i = 1, 2$*, above their steady state values* z^{∞} *and* u^{∞} *and their gradual decrease until the steady state values are reached. However, the sign of* $d\hat{\eta}/dt$ *remains undetermined.*

The proof is shown in the appendix. Suppose that the pollution stock is greater than its steady state value and we have to implement a pollution abatement policy. When the indirect effect of the pollution stock in the marginal cost of abatement is of a minor order, either because the cross derivative $k_{\eta s}$ is low or because the abatement cost function is highly convex with respect to the level of abatement ($k_{\eta\eta}$ high), the marginal damage and marginal cost of pollution stock increases more rapidly that the increase in the additional abatement evaluated in terms of the marginal cost, that is, $d_{ss} + k_{ss} > |k_{\eta s}\frac{\partial\eta}{\partial s}|$. In this case, the initial abatement effort at the receptor η' is not sufficient to reduce the stock of pollution and it has to be reinforced with a large decrease in aggregate emissions implying a high control at the source, consequently it must be a decrease in input use. When the stock of pollution decreases towards s^{∞}, as it is characterized by a low decrease of abatement at the receptor, a long-run optimal pollution abatement policy involves a decrease in φ over time, allowing an increase in the aggregate emissions, that leads to an increase in the input use and the production activities along the optimal path. Therefore, an intertemporally and spatially optimal pollution abatement policy, for $s_0 > s^{\infty}$, can be characterized by choosing the levels of applied input initially below their steady-state values. As time passes, they increase until their steady-state values are reached. This case is depicted in *Figure 7*.

Figure 7

However, if the marginal abatement cost increases slowly with an increase in η ($k_{\eta\eta}$ close to zero), it is possible that the effect of the additional abatement overcomes the change in the marginal damage and abatement costs produced by an additional unit of s. In this case, the optimal intertemporal abatement policy is characterized by a high initial abatement effort at the receptor η'. Hence, the decrease in aggregate emissions z and therefore the decrease of u_i, $i = 1, 2$, towards the steady state values is done gradually. This case is illustrated in *Figure 8*. *Figure 8* shows that the optimal intertemporal abatement policy is smoother than in the former case.

Figure 8

Therefore, the increase in the environmental damage and abatement costs set a limit on the possibility of abating the pollution stock after it has been generated. In the case that the pollutant becomes very dangerous above certain levels, entailing a high convexity of the environmental damage function (e.g., chromium), the optimal policy will require a high control of the pollution stock at the source. The same situation would arise if the increase in pollution stock leads to an abatement cost that makes it prohibitively expensive.
An especial case may be originated when $k = k(s(t))\eta(t)$, that is, the abatement cost is proportional to the amount of abated pollution stock. As abatement enters lineally in the model, the restoration policy will be given either exclusively by decreasing emissions at the source or increasing abatement depending on what policy has the lowest cost. During the early stages, when the pollution stock is large, controlling pollution at the source is expected to be a dominant strategy, until pollution decreases sufficiently to diminish the unitary abatement cost below the marginal value of pollution, i.e. $k(s(t)) < V_z$, after this point in time it will be no source control and the pollution will be abated after it has been generated.

The pattern of adoption of the precision technology will also change over time as a decrease (increase) of shadow cost along the optimal path results in an increase (decrease) in the quasirent for both activities. In the case where the quasirents of different activities intersect, this increase will also lead to a different optimal technology adoption. In the case where quasirents of the different activities do not intersect along the entire optimal path, the optimal technology choice pattern does not change at all. However, the quasirent of the different technologies might intersect along some part of the optimal path, leading to a change in the optimal technology choice pattern during this time and constancy otherwise.

5 The Optimal spatial and intertemporal Policies

5.1 Private Optimum with Input and Technology Policies

The social optimum, characterized by the equations (2) - (4) however, is not equivalent to the private optimum since the producer does not consider the externality. Her decision problem is simply given by

$$V(P) \equiv \max_{x_i, u_i} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} (ph_i(\epsilon) f(\beta_i u_i) - cu_i - I_i) x_i \Big) \, d\epsilon \qquad (P1)$$

subject to

$$x_i(\epsilon) \geq 0, \; i = 1, 2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon), \quad u_i(\epsilon) \geq 0, \; i = 1, 2$$

The private behavior will lead to an amount of aggregate emissions above its optimal social level. A first-best policy would call for a tax on individual emissions contributing to environmental damage at the receptor. However, individual emissions cannot be observed due to high costs or technical infeasibility (Kopman and Smith, 1993) and policy makers must rely on other policy measures which have to be observable and correlate as close as possible to the individual emissions (Braden and Segerson, 1993). These selection criteria are met by input taxes together with technology policies, provided they are based on site specific information. Since the pollution function is linear with respect to the fixed asset, the following proposition establishes the policies that lead to an optimal input use and technology adoption.

Proposition 5 *: Provided that input use and technology choices can be observed in each unit with quality* ϵ*, an optimal policy can be obtained by*

1. *a spatially differentiated input tax* τ_i*,* $i = 1, 2$*, given by* $\tau_i = \lambda^* \alpha_i g_{u_i}(u_i^*)$*,* $i = 1, 2$*, together with*

2. *a spatially differentiated technology subsidy or tax per unit of asset* σ_i*,* $i = 1, 2$*, given by* $\sigma_i = -\tau_i u_i^* + \lambda^* \alpha_i g(u_i^*) \gtreqless 0$.

Proof: The private decision problem where we have an input tax and a technology subsidy / tax yields $\max_{x_i, u_i} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} (p h_i(\epsilon) f(\beta_i u_i) - c u_i - I_i) x_i \Big) - \Big(\sum_{i=1}^{2} (\tau_i u_i x_i + \sigma_i x_i) \Big) d\epsilon$. Analyzing the necessary conditions of the problem we can see that the private optimum coincides with the social optimum given by equations (2) to (4), thus the input tax τ_i, $i = 1, 2$, together with the technology subsidy or tax σ_i, $i = 1, 2$, establishes the spatially differentiated optimal input-use and technology adoption in every location ϵ. ■

An input tax alone, however, is not sufficient to achieve the social optimum since it only establishes equation (2) but not equation (3), that is, the introduction of a tax on the intensive margin causes a distortion on the extensive margin, thus the social optimum is not realized. To establish the efficient allocation of technologies, the input tax needs to be complemented by a technology policy. The sign of σ_i, $i = 1, 2$, determines if we have a technology subsidy or tax. In the case where it is positive we have a technology tax, and if it is negative we have in fact a subsidy. Substituting the value of the spatially differentiated input tax τ_i, $i = 1, 2$, into σ_i, $i = 1, 2$, we obtain:

$$\sigma_i = \lambda^* \alpha_i \big(g(u_i^*) - g_{u_i} u_i^* \big) \gtreqless 0 \qquad (13)$$

As it has been shown in a proposition by Goetz and Zilberman (manuscript), when the marginal contribution of applied input to pollution is increasing, that is $g(u_i)$ is convex, σ_i, $i = 1, 2$, is negative and we have a technology subsidy, otherwise σ_i, $i = 1, 2$, has to be a technology tax. In our model, the emission function is convex, thus, the input taxes need to be complemented by technology subsidies.

The introduction of an input tax induce the firm to substitute the variable input by the fixed asset, for example increasing the number of cultivated hectares by a farm, that in turn affects the amount of pollution generated. As the emission function is convex, the pollution expenditures per unit of asset ($\tau_i u_i^* = \lambda^* \alpha_i g_{u_i} u_i^*$) are higher than the shadow value of the emissions per unit of asset ($\lambda_i \alpha_i g(u_i^*)$), thus we need to apply a technology subsidy equal to the difference between the pollution expenditures and its shadow cost per unit of asset.

The input and technology policies depend on the quality of the asset and on time. The employment of information on asset qualities allows one to target specific units so that input and technology policies based on the emissions of pollutant at the receptor associated with a particular technology can be adjusted to the potential emissions characteristic of the unit. Moreover, the technology and input use are easy to monitor so that the policies can be enforced in practice as well. These taxes are also adjusted over time according to the shadow cost of the pollutant which varies according to the development of the stock of pollutant over time. To see how should evolve these policies over time, we analyze the steady state equilibrium that would be reached when the producing units do not consider the pollution they cause and we compare it with the social steady state. The private intertemporal equilibrium will be given by

$$\max_{\eta(t)} \int_0^\infty exp^{-\delta t} \Big(V(P) - d(s(t)) - k(\eta(t), s(t)) \Big)\, dt \qquad (P2)$$

subject to

$$\begin{aligned} \dot{s}(t) &= z^P - \eta(t) - \zeta\, s(t), \\ & s(0) = s_0, \quad \eta(t) \leq s(t). \end{aligned}$$

where z^P is the private level of aggregate emissions. In the private equilibrium the amount of aggregate emissions is not under the social planner control and therefore the social planner is only able to determine the amount of pollution abated at the receptor to decrease pollution stock. *Figures 9* and *10* show the difference between the steady state equilibrium that would be reached in this case and the social steady state, for the two analized cases, and they also illustrate the adjustment path.

Figures 9 and 10

Figures 9 and *10* suggest that the convexity of damage and abatement cost functions have important implications in the intertemporal policies. The difference between the private and social pollution stock in the steady state in *figure 9* is lower than in *figure 10.* This is due to the higher convexity of the damage and/or abatement costs. Moreover, when the abatement cost is highly convex in the level of abatement (*figure 9*) there must be a large decrease in aggregate emissions, implying a substantial increase in the taxes in the first stages of the abatement process, leading to a decrease in the resource use. This involves a high control at the source level accompanied with a decrease in the level of abatement at the receptor. In contrast, when marginal abatement and damage costs vary slightly (*figure 10*), the intertemporal taxes can be gradually adjusted. *Figures 9* and *10* also show the importance of an early intervention of the policy makers to control the stock of pollution since an increase in the difference between private and social pollution stock requires also an increase in the severity of the intervention. On the contrary, if the difference between private and social pollution stock is small, the policy measures can be less harshly implemented and relaxed as we approach to the steady state. Likewise, since producers do not gain from an improved environmental quality, it will be easier to apply an early policy than a late but draconian one.

6 Summary and Conclusions

This paper presents a modelling approach for the socially optimal management of an accumulating pollution stock when producers are heterogeneous. This proposal is analyzed in a general framework that can be implemented in different environments, like in the problem of water logging, pesticide resistance or carbon emissions among others. The optimal intertemporal pollution abatement policy is analyzed considering heterogeneity conditions. A static solution for the production subsystem is determined in the first stage, where it is shown that the optimal level of input use per unit of asset increases with an increase in the quality of the fixed asset. Moreover, it is determined the optimal technology choice pattern given the asset quality and the range of qualities that will be employed for environmental protection.

The socially optimal intertemporal equilibrium is determined in the second stage. It is illustrated that the optimal intertemporal change in the shadow price towards the long-run equilibrium depends on the convexity of the abatement cost and damage functions. Therefore, these functions establish the link between the short-run and long-run input demand functions, determining the optimal mix of controlling pollution at the source versus abatement at the receptor.

Due to the presence of an externality the private net benefit maximizing strategy does not correspond to the social one. Thus, environmental policies in form input taxes and technology taxes or subsidies are proposed which induce site specific responses rather than uniform responses by taking account of the vulnerability of the unit with respect to emissions for a given technology. In particular, the temporal aspect of the regulation is of great importance, since it determines the degree of severity and the time schedule of the policy measures.

The economic problem is solved within a spatial and intertemporal framework based on a two stage approach. This sequential procedure enhances the analytical tractability of the solution more easily. The two stages are linked by the common shadow price allowing a relationship to form between the optimal short-run and long-run technology allocation. Most importantly it allows the necessary changes to transform a spatially optimal, yet static, environmental policy analysis to an intertemporally and spatially optimal policy.

Appendix

Utilizing Pontryagin's Maximum Principle, the current Hamiltonian of the optimal pollution restoration strategy (S) is given by

$$\begin{aligned} \mathcal{H} \;\equiv\; & \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(p h_i(\epsilon) f(\beta_i u_i(t,\epsilon)) - c u_i(t,\epsilon) - I_i\big) x_i(t,\epsilon)\Big)\, d\epsilon \\ & -\Big(d(s(t)) + k(\eta(t))\Big) \\ & -\mu\Big(\int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(t,\epsilon)) x_i(t,\epsilon)\Big)\, d\epsilon - \big(\eta(t) + \zeta\big) s(t)\Big). \end{aligned}$$

To facilitate the interpretations of the costate variable μ, it has been multiplied by minus one. In this way μ has a positive value. The arguments ϵ and t of the variables and the Lagrange multipliers will be suppressed to simplify the notation unless it is required for an unambiguous notation. Taking account of the constrains on the control variables leads to the Lagrangian: $\mathcal{L} \equiv \mathcal{H} + \omega_1 u_1 + \omega_2 u_2 + \omega_3 x_1 + \omega_4 x_2 + \omega_5(X - x_1 - x_2) + \omega_6 \eta + \omega_7(s - \eta)$, where $\omega_1, \ldots, \omega_7$ denote Lagrange multipliers. The solution of problem (S) has to satisfy the following necessary conditions stated in accordance with Theorem 1, page 276, Seierstad and Sydsæter (1987)

$$\begin{aligned} \mathcal{L}_{u_1} &\equiv (p h_1 \beta_1 f_{u_1} - c - \mu \alpha_1 g_{u_1}) x_1(t,\epsilon) + \omega_1 = 0, && \text{(A. 1)} \\ \mathcal{L}_{u_2} &\equiv (p h_2 \beta_2 f_{u_2} - c - \mu \alpha_2 g_{u_2}) x_2(t,\epsilon) + \omega_2 = 0, && \text{(A. 2)} \\ \mathcal{L}_{x_1} &\equiv p y_1 - c u_1(t,\epsilon) - I_1 - \mu \alpha_1 g(u_1(t,\epsilon)) + \omega_3 - \omega_5 = 0, && \text{(A. 3)} \\ \mathcal{L}_{x_2} &\equiv p y_2 - c u_2(t,\epsilon) - I_2 - \mu \alpha_2 g(u_2(t,\epsilon)) + \omega_4 - \omega_5 = 0, && \text{(A. 4)} \\ \mathcal{L}_{\eta} &\equiv -k_\eta + \mu + \omega_6 - \omega_7 = 0, && \text{(A. 5)} \\ \dot{\mu}(t) &= \delta\mu + \mathcal{H}_s = \mu(\delta + \zeta) - d_s - k_s + \omega_7, && \text{(A. 6)} \\ \dot{s}(t) &= \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i g(u_i(t,\epsilon)) x_i(t,\epsilon)\Big)\, d\epsilon - \eta(t) + \zeta s(t), \\ & \quad s(0) = s_0. && \text{(A. 7)} \end{aligned}$$

The analytical solution of the necessary conditions (A. 1) - (A. 7) is difficult. Thus, we rewrite the problem in the following way

$$\max_{u_i(t,\epsilon),\, x_i(t,\epsilon)} \int_0^\infty \exp^{-\delta t} A_1 \, dt + \max_{\eta(t)} \int_0^\infty \exp^{-\delta t} A_2 \, dt \qquad \text{(A. 8)}$$

subject to

$$\begin{aligned} \dot{s}(t) &= B_1 + B_2, && \text{(A. 9)} \\ & s(0) = s_0, \quad \eta(t) \le s(t), \quad u_i(t,\epsilon) \ge 0, \; i = 1,2, && \text{(A. 10)} \\ & x_i(t,\epsilon) \ge 0, \; i = 1,2, \quad x_1(t,\epsilon) + x_2(t,\epsilon) \le X(\epsilon). && \text{(A. 11)} \end{aligned}$$

where

$$
\begin{aligned}
A_1 &\equiv \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(ph_i(\epsilon) f(\beta_i u_i(t,\epsilon)) - cu_i(t,\epsilon) - I_i\big) x_i(t,\epsilon) \Big) d\epsilon \\
A_2 &\equiv -\Big(d(s(t)) + k(\eta(t), s(t))\Big) \\
B_1 &\equiv \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(t,\epsilon)) x_i(t,\epsilon) \Big)\, d\epsilon \\
B_2 &\equiv -\eta(t) - \zeta s(t),
\end{aligned}
$$

Relations and inequalities in (A. 10) and (A. 11) are block conditions (local constraints for subsystems) whereas equation (A. 9) gives the equation of motion for the dynamic system. According to the Kornai-Liptak decomposition principle (Sanders, 1964), we write a local problem of reduced dimensionality:

$$
\max_{u_i,\, x_i} \qquad A_1 - \lambda(z - B_1) \tag{S1}
$$

subject to

$$
B_1 = z \quad u_i \geq 0,\ i = 1,2, \quad x_i \geq 0,\ i = 1,2, \quad x_1 + x_2 \leq X(\epsilon).
$$

were λ is a Lagrange multiplier and z is a variable to be determined endogenously. In our model, z denotes the aggregate emissions over the entire range of ϵ, from ϵ_0 to ϵ_1 which accumulates over time.

That is, in the first stage we analyze the optimal solution over ϵ, given by

$$
\max_{u_i(\epsilon),\, x_i(\epsilon)} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(ph_i(\epsilon) f(\beta_i u_i(\epsilon)) - cu_i(\epsilon) - I_i\big) x_i(\epsilon) \Big)\, d\epsilon \tag{S1'}
$$

subject to

$$
\begin{aligned}
z &= \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(\epsilon)) x_i(\epsilon) \Big)\, d\epsilon, \\
& u_i(\epsilon) \geq 0,\ i = 1,2, \quad x_i(\epsilon) \geq 0,\ i = 1,2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon),
\end{aligned}
$$

To facilitate the interpretations of the costate variable λ, it has been multiplied by minus one. In this way λ has a positive value. The arguments ϵ and t of the variables and the Lagrange multipliers will be suppressed to simplify the notation unless it is required for an unambiguous notation.

Taking account of the constrains on the control variables leads to the Lagrangian

$$
\begin{aligned}
\mathcal{L}1 \;\; &\equiv \;\; \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(ph_i f(\beta_i u_i) - cu_i - I_i \big) x_i \Big) \, d\epsilon \\
&+\lambda \Big(z - \int_{\epsilon_0}^{\epsilon_1} \big(\sum_{i=1}^{2} \alpha_i g(u_i) x_i \big) \, d\epsilon \Big) \\
&+\upsilon_1 u_1 + \upsilon_2 u_2 + \upsilon_3 x_1 + \upsilon_4 x_2 + \upsilon_5 \big(X - x_1 - x_2 \big).
\end{aligned}
$$

The solution of problem (S1') has to satisfy the following necessary conditions stated in accordance with Theorem 1, page 276, Seierstad and Sydsæter (1987)

$$
\begin{aligned}
\mathcal{L}1_{u_1} \;\; &\equiv \;\; (ph_1\beta_1 f_{u_1} - c - \lambda\alpha_1 g_{u_1}) x_1 + \upsilon_1 = 0, && \text{(A. 12)} \\
\mathcal{L}1_{u_2} \;\; &\equiv \;\; (ph_2\beta_2 f_{u_2} - c - \lambda\alpha_2 g_{u_2}) x_2 + \upsilon_2 = 0, && \text{(A. 13)} \\
\mathcal{L}1_{x_1} \;\; &\equiv \;\; py_1 - cu_1 - I_1 - \lambda\alpha_1 g(u_1) + \upsilon_3 - \upsilon_5 = 0, && \text{(A. 14)} \\
\mathcal{L}1_{x_2} \;\; &\equiv \;\; py_2 - cu_2 - I_2 - \lambda\alpha_2 g(u_2) + \upsilon_4 - \upsilon_5 = 0, && \text{(A. 15)} \\
\mathcal{L}1_{\lambda} \;\; &\equiv \;\; z - \int_{\epsilon_0}^{\epsilon_1} \big(\sum_{i=1}^{2} \alpha_i g(u_i) x_i \big) \, d\epsilon = 0. && \text{(A. 16)}
\end{aligned}
$$

Let $u_i^*(z)), x_i^*(z)$, $i = 1, 2,$, be the optimal solution of the problem (S1), and let V(z) be the value function of the maximization problem, then, the coordinating problem takes the form

$$
\max_{z(t)} \int_0^\infty \exp^{-\delta t} V(z(t)) \, dt + \max_{\eta(t)} \int_0^\infty \exp^{-\delta t} A_2(\eta(t), s(t)) \, dt \qquad (S2)
$$

subject to

$$
\dot{s}(t) = z(t) - B_2(\eta(t), s(t)), \quad s(0) = s_0, \quad 0 \le \eta(t) \le s(t),
$$

Rewriting terms,

$$
\max_{z(t),\, \eta(t)} \int_0^\infty exp^{-\delta t} \Big(V(z(t)) - d(s(t)) - k(\eta(t), s(t)) \Big) \, dt \qquad (S2')
$$

subject to

$$
\dot{s}(t) = z(t) - \eta(t) - \zeta \, s(t), \quad s(0) = s_0, \quad 0 \le \eta(t) \le s(t).
$$

The current value Hamiltonian of the second stage is given by: $\mathcal{H}2 = V(z(t)) - d(s(t)) - k(\eta(t), s(t)) - \varphi\Big(z(t) - \eta(t) - \zeta s(t)\Big)$, where φ denotes the costate variable. The first order conditions read as

$$
\begin{aligned}
\mathcal{L}2_z &\equiv V_z - \varphi = 0, && \text{(A. 17)}\\
\mathcal{L}2_\eta &\equiv -k_\eta + \varphi + \upsilon_6 - \upsilon_7 = 0, && \text{(A. 18)}\\
\dot{\varphi} &= \delta\varphi + \mathcal{H}2_s = \big(\delta + \zeta\big)\varphi - d_s - k_s + \upsilon_6, && \text{(A. 19)}\\
\dot{s} &= z - \eta - \zeta s, \qquad s(0) = s_0. && \text{(A. 20)}
\end{aligned}
$$

Assume we find the optimum of the problems (S1) and (S2). That is we find the functions u_i^*, x_i^*, $i = 1, 2$, z^* and η^* that solve the FOC (A. 12) to (A. 20). Given that

- equations (A. 12) to (A. 15) of problem (S1) are parallel to equations (A. 1) to (A. 4) of the main problem,
- equation (A. 18) of problem (S2) is equivalent to equation (A. 5) of problem (S),
- equation (A. 16) is the definition of aggregate emissions that we introduced, thus its substitution into equation (A. 20) leads to equation (A. 7) of problem (S),
- and equation (A. 17) relates the two stages. The change in the value function of the static problem (S1) given by a marginal increment of the aggregate emissions z is given by $V_z = dV/dz = \partial\mathcal{L}1/\partial z = \lambda$, where we made use of the envelope theorem, thus we obtain that $\lambda^*(t) = \varphi^*(t)$. That is, the shadow cost of the pollution stock φ is equal to the shadow cost of the aggregate emissions in the static problem λ. This link allows us to relate the optimal abatement policy over ϵ with the optimal intertemporal abatement policy,

we can conclude that this functions will also maximize the problem (S), with the Lagrangian multipliers $\mu^*(t) = \lambda^*(t) = \varphi^*(t)$ and $\omega_1^*, \dots, \omega_7^* = \upsilon_1^*, \dots, \upsilon_7^*$ respectively. ∎

To find the effect of the change in the asset quality in the intensity of applied input we differentiate equations (A. 12) and (A. 13) with respect to ϵ and solve for $\partial u_i/\partial\epsilon$ obtaining:

$$
\frac{\partial u_i}{\partial \epsilon} = \frac{-(ph_i'\beta_i f_{u_i} - \lambda\alpha_i' g_{u_i})}{ph_i\beta_i^2 f_{u_iu_i} - \lambda\alpha_i g_{u_iu_i}} > 0. \qquad \text{(A. 21)}
$$

The changes in the allocation of the technologies are determined by differentiating equations (A. 14) and (A. 15) with respect to ϵ:

$$
\Pi_{i_\epsilon}^* \equiv \mathcal{H}1_{x_i\epsilon} = [ph_i' f(\beta_i u_i)] - [\lambda\alpha_i' g(u_i)] > 0, \qquad \text{(A. 22)}
$$

where the first term in brackets presents the value of the change in production per unit of asset due to an increase in the asset productivity, and the change in the emissions per unit of asset that reach the receptor is measured by the second term in brackets. Since $h_i' > 0$, $f(\beta_i u_i) > 0$ and $\lambda\alpha_i' g(u_i) < 0$, the quasirent of activity i, $i = 1, 2$, is upward sloping with an increase in ϵ.

The concavity of the quasirent is shown in the second derivative, given by

$$\Pi^*_{i\epsilon\epsilon} = [ph''_i f(\beta_i u_i) - \lambda\alpha''_i g(u_i)] - [(ph'_i\beta_i f_{u_i} - \lambda\alpha'_i g_{u_i})\frac{\partial u_i}{\partial\epsilon}] \gtreqless 0, \qquad (A.\,23)$$

In order to analyze the effect of the change in the shadow cost or in the stock of pollution over the aggregate emissions (control at the source) and abatement at the receptor we conduct a comparative static analysis. The sign of $\partial\hat{z}/\partial\varphi$ and $\partial\hat{\eta}/\partial\varphi$ can be determined by solving the first order equations (A. 17) and (A. 18) for $z = \hat{z}(\varphi, s)$ and $\eta = \hat{\eta}(\varphi, s)$. Hence by the implicit function theorem, we obtain

$$\begin{pmatrix} \mathcal{L}2_{zz} & \mathcal{L}2_{z\eta} \\ \mathcal{L}2_{\eta z} & \mathcal{L}2_{\eta\eta} \end{pmatrix} \begin{pmatrix} \frac{\partial\hat{z}}{\partial\varphi} & \frac{\partial\hat{z}}{\partial s} \\ \frac{\partial\hat{\eta}}{\partial\varphi} & \frac{\partial\hat{\eta}}{\partial s} \end{pmatrix} + \begin{pmatrix} \mathcal{L}2_{z\varphi} & \mathcal{L}2_{zs} \\ \mathcal{L}2_{\eta\varphi} & \mathcal{L}2_{\eta s} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \qquad (A.\,24)$$

Thus, the application of Cramer's rule yields that

$$\frac{\partial\hat{z}}{\partial\varphi} = \frac{1}{V_{zz}} \leq 0, \quad \frac{\partial\hat{z}}{\partial s} = 0, \quad \frac{\partial\hat{\eta}}{\partial\varphi} = \frac{1}{k_{\eta\eta}} \geq 0, \quad \frac{\partial\hat{\eta}}{\partial s} = -\frac{k_{\eta s}}{k_{\eta\eta}} \geq 0. \qquad (A.\,25)$$

The implicit function theorem is also used to calculate the elements of the Jacobian matrix evaluated at the steady state equilibrium with $\dot{\varphi} = \dot{s} = 0$, leading to

$$\tilde{J} = \begin{pmatrix} \frac{\partial\dot{\varphi}}{\partial\varphi} = \delta + \zeta - k_{\eta s}\frac{\partial\eta}{\partial\varphi} > 0 & \frac{\partial\dot{\varphi}}{\partial s} = -k_{\eta s}\frac{\partial\eta}{\partial s} - d_{ss} - k_{ss} \gtreqless 0 \\ \frac{\partial\dot{s}}{\partial\varphi} = \frac{\partial z}{\partial\varphi} - \frac{\partial\eta}{\partial\varphi} < 0 & \frac{\partial\dot{s}}{\partial s} = -\zeta - \frac{\partial\eta}{\partial s} < 0 \end{pmatrix}. \qquad (A.\,26)$$

To find the optimal intertemporal path of $z(t)$ and $\eta(t)$, we find the total differentiation with respect to time given by

$$\frac{d\hat{z}}{dt} = \frac{\partial\hat{z}}{\partial\varphi}\frac{d\varphi}{dt} + \frac{\partial\hat{z}}{\partial s}\frac{ds}{dt} = \frac{1}{V_{zz}}\frac{d\varphi}{dt}, \qquad (A.\,27)$$

$$\frac{d\hat{\eta}}{dt} = \frac{\partial\hat{\eta}}{\partial\varphi}\frac{d\varphi}{dt} + \frac{\partial\hat{\eta}}{\partial s}\frac{ds}{dt} = \frac{1}{k_{\eta\eta}}\frac{d\varphi}{dt} - \frac{k_{\eta s}}{k_{\eta\eta}}\frac{ds}{dt}. \qquad (A.\,28)$$

Finally, we conduct a comparative static analysis to analyze the effect of the change in the shadow cost over the input use. Since neither V nor λ depend on ϵ, we assume that the technologies are located optimally and the amount of pollution is chosen optimally, that is we are moving along the optimal path, the sign of $\partial u_i^*/\partial\lambda$ can be determined by solving the first order equations (A. 12) and (A. 13) for $u_i = u_i^*(\lambda)$, $i = 1, 2$. Hence by the implicit function theorem, we obtain

$$\begin{pmatrix} \mathcal{L}1_{u_1u_1} & \mathcal{L}1_{u_1u_2} \\ \mathcal{L}1_{u_2u_1} & \mathcal{L}1_{u_2u_2} \end{pmatrix} \begin{pmatrix} \frac{\partial u_1^*}{\partial\lambda} \\ \frac{\partial u_2^*}{\partial\lambda} \end{pmatrix} + \begin{pmatrix} \mathcal{L}1_{u_1\lambda} \\ \mathcal{L}1_{u_2\lambda} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \qquad (A.\,29)$$

As before, we apply Cramer's rule that yields

$$\frac{\partial u_1^*}{\partial\lambda} = \frac{\alpha_1 g_{u_1}}{ph_1 f_{u_1u_1} - \lambda\alpha_1 g_{u_1u_1}} < 0, \quad \frac{\partial u_2^*}{\partial\lambda} = \frac{\alpha_2 g_{u_2}}{ph_2 f_{u_2u_2} - \lambda\alpha_2 g_{u_2u_2}} < 0. \qquad (A.\,30)$$

Using equations (A. 27), (A. 28) and (A. 30) and the fact that $\lambda = \varphi$ and that the slope of the stable path is determined by the sign of $\partial\dot{\varphi}/\partial s$, proposition 4 can be verified.

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Table 1: Examples of Environmental Problems in Agriculture

Type of Problem	Decision Variables			State Variable
	Technology	Products	Inputs	
Soil erosion and siltation of rivers, canals, reservoirs	abatement technology capital - terracing - machinery with special tires - dams cultivation techniques - minimum tillage - conventional tillage - working along isohight lines	crops cover crops	intensity of tillage	sedimentation layer
Water logging	irrigation technologies - furrow irrigation - drip irrigation - sprinkler irrigation	crops	water	height of the water table concentration of salt
Phosphorus and Nitrogen emissions	cultivation techniques - N min method of fertilization - frequency of N applications - application machinery type of fertilizer - organic - inorganic	crops number and types of animals catch crops	Nitrogen and Phosphorus	concentration of Nitrogen or Phosphorus
GHG emissions	cultivation techniques - minimum tillage - conventional tillage	agriculture - cropland - permanent grass land afforestation	intensity of tillage	amount of Carbon sequestrated