Defining sustainability objectives

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January 22, 2009

Abstract

Two of the challenges of thinking sustainability are how to deal with potentially conflicting issues and how to ensure intergenerational equity. In practice, policymakers define sustainability objectives by setting thresholds that act as constraints on indicators. When defining a specific objective, they usually do not take into account either potential conflicts with other objectives or the difficulty to achieve all of them over time. In this paper, we propose an approach that defines sustainability objectives represented by a set of constraints on indicators and their associated thresholds. This approach meets the challenges of sustainability because objectives are defined such that all the constraints can be satisfied at all times. The thresholds are interpreted as minimal rights to be guaranteed to all generations, in a Rawlsian equity perspective. To define them, we have developed a criterion, which is, from a mathematical point of view, a "generalized" maximin. Applying the criterion is a two-step process. Firstly, the set of achievable objectives, given the endowment of the economy, is defined, revealing the necessary trade-offs between them. Secondly, a static optimization of sustainability preferences on that set results in the proposed definition of sustainability objectives. We illustrate this approach by applying it to a canonical model often used to investigate sustainability issues (Dasgupta-Heal-Solow model; Review of Economic Studies 1974). We emphasize the relevance of this approach because it rationalizes the practice of using indicators to deal with sustainability in terms of the given challenges. We also discuss how to apply our approach to real sustainability issues.

Key-words: sustainability, indicators, intergenerational equity, criterion, minimal rights, viability.

JEL Classification: Q01, Q32, O13, C61.

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1 Introduction

On the occasion of the fortieth anniversary of Resources For the Future, in a lecture entitled "An almost practical step toward sustainability," Robert Solow pointed out that

> *If sustainability means anything more than a vague emotional commitment, it must require that something be conserved for the very long run. It is very important to understand what that thing is.* (Solow, 1993, p.167-168)

Indeed, if sustainability involves the conservation of "resources," broadly speaking, for future generations, not only do we need to determine what should be conserved, but also why. In other words, it is also important to thoroughly think about how sustainability objectives are defined.

Economic theory approaches the definition of sustainability objectives by first determining so-called *sustainability criteria*, resulting in intertemporal economic trajectories (Dixit et al., 1980; Beltratti et al., 1995; Cairns and Long, 2006). These criteria are evaluated with respect to the way they deal with the two main challenges of sustainable development: taking into account potentially conflicting issues, such as environmental concerns and economic development, on the one hand and intergenerational equity on the other hand (Heal, 1998). The former challenge is often tackled by adding related arguments in the utility function (Krautkraemer, 1985). From a general point of view, as sustainability has to encompass various issues, a comprehensive utility function that reflects all of the sustainability objectives must be defined (Hediger, 2000), and the usual way to do so is to consider that the utility function depends on all stocks and decisions. Applying such an approach to real-world issues may be difficult as it requires solving a multiobjective dynamic optimization problem. In practice, optimal control problems with multi-attribute utility functions often require an oversimplification of the objective function. For example, in order to explicitly represent the trade-offs between the conflicting objectives, the utility function can be linearized by weighting all the criteria. Another way to tackle this first challenge is to impose exogenous sustainability constraints. For example, strong sustainability is based on a constraint approach to ensure the preservation of critical natural resources (Daly, 1974). This approach raises the issue of the definition of the level of such constraints.

The second main challenge, that is to say the intergenerational equity issue, has been deeply investigated. A widely recognized number of studies based on an axiomatic approach show that there is no preference rule over infinite streams of utilities that results in an explicit criterion satisfying efficiency properties *and* treating all generations equally, i.e., with anonymity (Koopmans, 1960; Diamond, 1965; Cass, 1965; von Weizsäcker, 1980; Epstein, 1986a,b; Basu and Mitra, 2003; Fleurbaey and Michel, 2003; Sakai, 2006). Svensson (1980), however, argues the existence of non-explicit criteria, which makes them impossible to apply. Chichilnisky

(1996) proposes a very interesting approach defining a (family of) criterion satisfying a set of axioms representing weakened intergenerational equity requirements. But her criterion is not easy to apply, and has no solution in some problems (Heal, 1998).

Altogether, these economic approaches can be qualified as "top-down": they start from theory and general criteria, which are sometimes difficult to apply in practice, and result in complex management rules associated with an optimal intertemporal trajectory.

In practice, sustainability is handled by using lists of indicators that reflect several issues, often classified for sake of simplicity into three categories: economical, environmental and social (UN, 2001). However, an indicator by itself is neither a policy nor an objective; it is merely a measurement of something. Setting thresholds that act as constraints on these indicators to define sustainability objectives draws the boundaries within which the economy should stay, and defines the minimum standards the present generation should pass on to future generations. The question remains: Do policymakers, when defining a specific objective, take into account potential conflicts with other objectives and the difficulty to achieve all of them over time? If not, this practice is not consistent with the given concept of sustainability.

Knowing that sustainability indicators are the basis of policy-making, their use can be viewed as the "bottom line" of sustainability. This given favors a "bottom-up" approach. Starting from indicators, is it possible to develop a criterion that would define, in a rational way, sustainability objectives as thresholds acting as constraints on those indicators? What would the theoretical implications of such an approach be?

In this paper, we propose an approach that defines sustainability objectives using sustainability *indicators*. *Sustainability objectives* are represented by a set of constraints on indicators, and their associated thresholds. Defining sustainable objectives thus consists in defining what these thresholds should be. A *sustainable development path* is defined as an economic trajectory for which all the sustainability objectives are achieved at all times. We interpret such sustainability objectives as minimal rights to be guaranteed to all generations, extending the Rawls' principle of justice (Rawls, 1971) to intergenerational issues. We assume the existence of a complete *preference* ordering on thresholds levels, and propose a *criterion* to define optimal sustainability objectives, in terms of thresholds, given economic endowments.

Applying the criterion is a two step process. Firstly, the set of objectives that are achievable for all generations, given the economic endowments, is defined. This reveals the necessary trade-offs between sustainability objectives in a dynamic perspective. Secondly, we define the optimal sustainability objectives among that set, according to sustainability preferences. Such a static optimization problem is much simpler to solve than dynamic optimization problems associated with the usual criteria.

We illustrate our general approach by applying it to a canonical model (the so-called Dasgupta-Heal-Solow model) often used to investigate sustainability issues (Dasgupta and Heal, 1974; Solow, 1974; Heal, 1998). It is an intertemporal resource allocation model with a manufactured capital stock and a non renewable natural resource.

We discuss in what ways our approach can be viewed as a practical step toward sustainability, and how it can be helpful to rationalize the practice of using indicators to deal with sustainability, in the light of the given challenges. Moreover, we explain how such an approach could be applied to "real case" sustainability problems, and emphasize that adequate numerical and algorithmic methods exist for application studies.

The remainder of the paper is organized as follows. Our general approach to defining sustainability objectives is presented in section 2. An application to the canonical Dasgupta-Heal-Solow model is provided in section 3. The approach and its applicability are discussed in section 4. We conclude in section 5 on the relevance of our approach and potential developments.

2 An approach to defining sustainability objectives

Dynamic economic model

Consider an economy with n capital stocks represented by the vector $X \in \mathbb{X} \subseteq \mathbb{R}^n$. Each capital stock can either be reproducible man-made capital or a natural resource (renewable or not). Let us define the decision vector $u \in \mathbb{U} \subseteq \mathbb{R}^m$. Each component of u can be interpreted as a consumption, resource extraction, or investment modifying the stocks. In a continuous time dynamic framework, we represent the dynamics of the economy (either capital dynamics, production functions, or natural resources dynamics) by function F. Denoting $\dot{X} \equiv \frac{dX}{dt}$, the economic state X evolves according to the following equation.

$$\dot{X} = F(X, u) \tag{1}$$

Let the endowments of the economy be denoted by the initial stock levels $X(0) = X_0$.

2.1 Sustainability objectives

We assume that sustainability has to encompass a given finite number I of (potentially conflicting) issues of a different nature (economical, environmental, and social) in an intertemporal framework.

Sustainability issues are addressed in the real world using indicators. These indicators represent some *characteristics* of the economy. The sustainability of an economy is thus not described directly by stocks or decisions, but rather by various characteristics defined as functions of capital stocks and decisions.

Sustainability indicators

To address sustainability, we consider a finite number I of *sustainability characteristics*, each one being associated to one of the sustainability issues.[1] Each characteristic can encompass economical, environmental, and/or social dimensions. These characteristics depend on the state of the economy and on decisions. We consider I *indicators* that measure each of the sustainability characteristics for given economic state and decisions. We define those indicators as follows.

Definition 1 *A* **sustainability indicator** *is a function* $\mathcal{C}_i : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$, *that provides a measure* $\mathcal{C}_i(X, u)$ *of sustainability characteristic* i *in economic state* X *when decisions* u *are applied.*

We postulate that, at a given time t, the indicators depend only on the stocks and decisions at that time, i.e., $\mathcal{C}_{i=1,\dots,I}(X(t), u(t))$.[2] Moreover, we stipulate that sustainability indicators are defined in such a way that large values are preferred to small ones. This means that indicators representing "bads" (pollution...) are represented by negative values. This latter assumption implies that, broadly speaking, the higher sustainability characteristics, the better.

Sustainability objectives seen as constraints

We associate a minimal threshold $\underline{c}_i$ to each indicator. We specify that a sustainability objective is to maintain the level of the sustainability characteristic i above the threshold $\underline{c}_i$. It leads to the sustainability constraint $\mathcal{C}_i \geq \underline{c}_i$ on each indicator.

Definition 2 *A* **sustainability objective** *is an indicator* $\mathcal{C}_i(X, u)$ *associated with a threshold* $\underline{c}_i$, *resulting in a sustainability constraint* $\mathcal{C}_i \geq \underline{c}_i$.

From that definition, given a set of sustainability indicators, defining sustainability objectives consists in defining thresholds for the I sustainability indicators. An important detail at this stage is that the thresholds $\underline{c}_{i=1,\dots,I}$ are not defined exogenously, and that the purpose of the analysis is to define them (define sustainability objectives). We are interested in a way *a)* to examine necessary trade-offs between conflicting sustainability objectives (how much one can increase an objective w.r.t. the others), and *b)* to represent preferences over these objectives (if one could increase an objective, which one would be preferred).

Sustainable development paths

To account for intergenerational equity, sustainability constraints must be satisfied at all times. It means that sustainability objectives represent the rights of any generation to benefit from achievement of all objectives $i = 1, \dots, I$ above thresholds $\underline{c}_{i=1,\dots,I}$. Using the previous definitions, we define a sustainable development trajectory as follows:

[1] Each characteristic can be interpreted as a sustainability "good".

[2] As a remark, note that the generation living at time t benefits actually from sustainability characteristics $\mathcal{C}_{i=1,\dots,I}(X(t), u(t))$.

Definition 3 *A economic trajectory* $(X(.), u(.))$ *is sustainable with respect to objectives* $\underline{c}_{i=1,\dots,I}$*, if at all times* t*,* $\mathcal{C}_i(X(t), u(t)) \geq \underline{c}_i$*, for all* $i = 1, \dots, I$*.*

Defined this way, sustainability objectives are constraints to be satisfied by the economy at all times. If thresholds $\underline{c}_{i=1,\dots,I}$ are known, the problem of defining sustainable trajectories is a *viability* problem. The purpose of a viability problem is to study the consistency between the dynamics of the economy and the sustainability objectives, and to determine the economic decisions which make it possible to achieve all the objectives at all time (see the description of the links between the viability approach and the sustainability issue in the appendix). In that sense, it does not allow trade-offs, neither between objectives (all constraints must be satisfied), nor between time periods (the constraints must be satisfied at all times). It means that all the objectives and all the generations are treated the same way, and that if one (or more) of the objectives is not satisfied at some time, the economy is not sustainable.

A question that arises now is: How are we to go about defining the thresholds that would characterize sustainability objectives?

2.2 Maximizing minimal rights for sustainability

In this section, we propose a way to define sustainability objectives, interpreted as minimal rights to be guaranteed to all generations, and represented by constraints on indicators and associated thresholds.

Preferences

If sustainability does not depend directly on economic state and decisions but on *characteristics* of the economic path, sustainability preferences must depend on these characteristics, i.e., on sustainability indicators. In particular, sustainability preferences should not be defined by a general utility function $U(X, u)$. From that point of view, sustainability indicators are to our approach the same as Lancaster's consumption *properties* or *characteristics* (Lancaster, 1966), in the sense that they are defined as functions of usual economic elements such as consumption or capital (and thus economic state and decisions). In Lancaster's analysis of consumption, those characteristics are the arguments of the preference functions. The same could be done in our sustainability issue, and preferences could depend on sustainability indicators. However, in order to take into account the intergenerational equity concerns that are specific to the sustainability issue, sustainability preferences should not depend on the actual level of the indicators, but on something reflecting the way the characteristics measured by these indicators are sustained. In our approach, intergenerational equity concern requires to offer the same minimal rights, defined as constraints on sustainability indicators, to all generations. We thus assume that sustainability preferences depend on the thresholds representing sustainability objectives. There is a twofold argument underlying this assumption. Firstly, it is consistent with our bottom-up approach starting from sustainability indicators and, as in practice, aiming at defining sustainability objectives as thresholds. Preferences thus depend on what has to be defined. Secondly, it is consistent with our approach that addresses the intergenerational equity issue in the viability framework of analysis, focusing on constraints that must be satisfied at all times.

We thus define a sustainability preference function[3] $\mathcal{P}$ which depends on the thresholds $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$. Moreover, under our assumption that sustainability indicators represent "goods", situations with higher sustainability objectives should be preferred.[4]

Definition 4 *We define a* **preference function** $\mathcal{P} : \mathbb{R}^I \mapsto \mathbb{R}$ *giving a real value* $\mathcal{P}(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$ *to any set of sustainability objectives* $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \in \mathbb{R}^I$. *The function* $\mathcal{P}$ *is increasing with respect to all its arguments, i.e., for any* i, *if* $c_i^a > c_i^b$ *then* $\mathcal{P}(\underline{c}_1, \ldots, c_i^a, \ldots, \underline{c}_I) \geq \mathcal{P}(\underline{c}_1, \ldots, c_i^b, \ldots, \underline{c}_I)$.

Note that $\mathcal{P}$ is not a measure of instantaneous welfare as it does not depend on the actual indicators levels $\mathcal{C}_i(X(t), u(t))$, but on the sustainability objectives $\underline{c}_i$.

The criterion

To define sustainability objectives, we introduce the following criterion, which defines the optimal sustainability objectives to be achieved at all times, with respect to the preference function $\mathcal{P}$.

Definition 5 *We define the problem:*

$$\begin{array}{rl} \max_{\underline{c}_i,\ i=1,\ldots,I} & \mathcal{P}(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \\ s.t \quad \mathcal{C}_i(X(t), u(t)) \geq \underline{c}_i & i = 1, \ldots, I \ ; \forall t \in \mathbb{R}^+ \\ \dot{X} = F(X(t), u(t)) & \\ X(0) = X_0 & \end{array} \tag{2}$$

To link our approach to the existing literature, we argue that, from the mathematical point of view, the criterion introduced in definition 5 is a generalized *maximin* (Solow, 1974; Cairns and Long, 2006). A maximin problem is actually a special case of eq.(2) when $I = 1$, considering only one sustainability objective. For example, if the objective is to sustain instantaneous utility $U(X(t), u(t))$, such

[3]We use a cardinal preference function in order to compare our approach to existing literature on sustainability criteria. However, the reader will see by the end that an ordinal preference relation on the sustainability objectives is sufficient to apply our framework.

[4]At that stage, we do not need further assumptions on the preferences. In particular case-studies, such as the one presented in Section 3, we should stipulate some properties of the preference function. Nevertheless, we will discuss in Section 4 how our framework can be used without having an explicit preference function.

a standard maximin "utilitarian" problem would have the form

$$\begin{array}{rl} \max_{\underline{U}} & \underline{U} \\ s.t & U(X(t),u(t)) \geq \underline{U} \quad \forall t \in \mathbb{R}^+ \\ & \dot{X} = F(X(t),u(t)) \\ & X(0) = X_0 \end{array} \tag{3}$$

It means that, in our framework, the preference function would be reduced to the simple one-argument linear form $\mathcal{P}(\underline{U}) = \underline{U}$. Moreover, all sustainability objectives would be encompassed in the utility function U and the threshold value $\underline{U}$.

In our approach, the sustainability objectives are taken as separate constraints, instead of being grouped in an utility function. The preference function is thus a combination of the thresholds, instead of the sole minimal utility, as in the standard maximin approach. We should give an interpretation of our approach with respect to the more usual maximin approach (with an utility) in the discussion part (section 4). But first, let us focus on the technical issue of solving the problem (2).

2.3 A methodology

As, from the mathematical point of view, our problem is a generalized maximin problem, a possibility to solve it would be to extend the direct approach proposed by Cairns and Long (2006). Their approach to solve problem (3) consists in developing a time autonomous optimization problem in which $\underline{U}$ is a control parameter. They introduce an adjoint variable for each of the n capital stocks (each adjoint variable being interpreted as the shadow value of the associated state variable), and another adjoint variable for the equity constraint $U(X(t),u(t)) \geq \underline{U}$. This last adjoint variable is interpreted as the shadow-value of equity. A complex dynamic optimization problem must then be solved.

Doing the same for our problem would require to consider I control parameters (all the $\underline{c}_i$) instead of a single one, and to introduce I adjoint variables on the respective sustainability constraints, being interpreted as the shadow-values of the satisfaction of each sustainability objective. It would result in a high dimensional dynamic optimization problem really hard to solve.

In order to avoid the resolution of such a difficult problem, we propose to split the problem in the two following steps.

Set of achievable sustainability objectives

First, we define the set of achievable sustainability objectives, given the initial state of the economy X_0.

$$\mathcal{S}(X_0) = \left\{ (\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) \;\middle|\; \begin{array}{l} \text{there exist decisions } u(.) \text{ such that} \\ \text{given } X_0 \text{ and the dynamics } \dot{X} = F(X,u) \\ \mathcal{C}_i(X(t),u(t)) \geq \underline{c}_i, \; i = 1,\ldots,I; \forall t \in \mathbb{R}^+ \end{array} \right\} \tag{4}$$

This first step[5] consists in defining the set of all combinations of objectives $\underline{c}_i$ that are achievable given the initial state of the economy X_0; Given the economic endowments X_0, it is possible to define a sustainable development trajectory, as introduced in definition 3, for any $(\underline{c}_1, \dots, \underline{c}_i, \dots, \underline{c}_I) \in \mathcal{S}(X_0)$. On the contrary, there is no sustainable trajectory from X_0 for any combination of sustainability objectives that is not in $\mathcal{S}(X_0)$.

This set of achievable sustainability objectives describes the necessary trade-offs between conflicting objectives. Once that trade-offs are described, it is easier to choose sustainability objectives among achievable ones.

Static optimization problem on preferences

Second, we solve the static optimization problem of determining from among the set of all feasible minimal rights the vector of minimal rights $(\underline{c}_1, \dots, \underline{c}_i, \dots, \underline{c}_I)$ that maximizes the proposed criterion,

$$\max_{(\underline{c}_1, \dots, \underline{c}_i, \dots, \underline{c}_I) \in \mathcal{S}(X_0)} \mathcal{P}(\underline{c}_1, \dots, \underline{c}_i, \dots, \underline{c}_I) \tag{5}$$

This second step is based on the preference function $\mathcal{P}$ that describes the preferential trade-offs between sustainability objectives.

3 Application to the Dasgupta-Heal-Solow (DHS) model

In order to describe the implication of such an approach, we apply it to the seminal two stock model by Dasgupta and Heal (1974) and Solow (1974).

A consumption-production economy with non-renewable resources

Consider an economy in which a non renewable resource x_t is extracted (at a rate r_t) and used with capital k_t to produce output. The production function is of the Cobb-Douglas form $k^\alpha r^\beta$, with $\alpha > \beta$. Output can be consumed (c_t) or invested ($\dot{k}$). Such a model has been studied in Dasgupta and Heal (1974) and Solow (1974), and is an useful stylized model for addressing the sustainability issue: the intertemporal allocation of the exhaustible resource, and the stream of consumption through time make intertemporal comparisons possible, in an intergenerational equity perspective (Krautkraemer, 1998). This model has been widely used in the sustainable development literature (Heal, 1998) to compare various sustainability criteria.

The dynamics are as follows.

$$\dot{k} = k_t^\alpha r_t^\beta - c_t, \tag{6}$$

$$\dot{x} = -r_t. \tag{7}$$

[5]From the mathematical point of view, this step is an extension of the viability approach described in the appendix. One has to define the set of all objectives $(\underline{c}_1, \dots, \underline{c}_i, \dots, \underline{c}_I)$ such that X_0 belongs to the viability kernel of the associated viability problem, and satisfies $V_{(\underline{c}_1, \dots, \underline{c}_i, \dots, \underline{c}_I)}(X_0) = 0$.

Sustainability objectives

To illustrate our approach, we will consider two sustainability objectives in that model. First, we want to sustain a consumption level $\underline{c}$. Second, we want to preserve a part of the stock $\underline{x}$. Sustaining consumption and preserving a part of the natural resource are the conventional objectives in this benchmark model (Heal, 1998). We thus consider the following sustainability objectives.

$$\begin{aligned} c_t &\geq \underline{c}, \quad (8) \\ x_t &\geq \underline{x}. \quad (9) \end{aligned}$$

The approach consists in defining the level of guaranteed consumption $\underline{c}$ and preserved resource stock $\underline{x}$ for sustainability.

Preferences

We introduce the preference function $\mathcal{P}$ on sustainability objectives, and assume that it satisfies the following conditions[6]

- $\underline{c} \in \mathbb{R}^+$; $\underline{x} \in \mathbb{R}^+$
- $\mathcal{P} : \mathbb{R}^+ \times \mathbb{R}^+ \mapsto \mathbb{R}^+$
- $\mathcal{P}_{\underline{c}} \geq 0$; $\mathcal{P}_{\underline{x}} \geq 0$; $\mathcal{P}_{\underline{c},\underline{x}} \leq 0$.

The criterion

We apply the criterion defined by eq.(2):

$$\max_{\underline{c},\underline{x}} \; \mathcal{P}(\underline{c}, \underline{x}) \quad (10)$$

subject to

$$\begin{aligned} (k_0, x_0) & \quad \textit{given} \\ c_t &\geq \underline{c} \\ x_t &\geq \underline{x} \\ \dot{k} &= k_t^\alpha r_t^\beta - c_t \\ \dot{x} &= -r_t \end{aligned}$$

Set of admissible sustainability objectives

Following the methodology presented in the previous section, we first define the set of objectives $(\underline{c}, \underline{x})$ that are achievable from the initial capital stocks of the economy (k_0, x_0). Doing that, we define the set of minimal rights that could be guaranteed to all generations:

[6] $\mathcal{P}_a$ denotes the partial derivative of $\mathcal{P}$ with respect to a. $\mathcal{P}_{a,b}$ stands for cross derivatives, and so on.

$$\mathcal{S}(k_0, x_0) = \left\{ (\underline{c}, \underline{x}) \;\middle|\; \begin{array}{l} \text{there exist paths } (k(.), x(.)) \text{ starting from } (k_0, x_0) \\ \text{that satisfy the constraints (8) and (9)} \end{array} \right\}.$$

To determine that set, we define the maximal sustainable consumption with respect to a preservation objective $\underline{x}$

$$c^+(k_0, x_0, \underline{x}) = \max\Big(c^\sharp \;\Big|\; \text{given } (k_0, x_0), \text{there exists } (c(.), r(.)) \\ \text{such that } \forall t \geq 0, c_t \geq c^\sharp \text{ and } x_t \geq \underline{x} \Big).$$

Solow (1974) first studied the sustainability of this economic model in the maximin framework. He proved, without considering a resource preservation objective, that the maximal sustainable consumption with respect to economic endowments (k_0, x_0) is

$$c^+(k_0, x_0, 0) = (1-\beta)\big(x_0(\alpha-\beta)\big)^{\frac{\beta}{1-\beta}} k_0^{\frac{\alpha-\beta}{1-\beta}}.$$

Martinet and Doyen (2007) described the relationship between consumption and preservation objectives in the same model. They examined in the viability framework the conditions for a minimal consumption $\underline{c}$ to be guaranteed when there is also a constraint on the preservation of the resource $\underline{x}$. They extended Solow's result by determining the maximal sustainable consumption given a preservation objective $\underline{x}$, which is

$$c^+(k_0, x_0, \underline{x}) = (1-\beta)\big((x_0 - \underline{x})(\alpha-\beta)\big)^{\frac{\beta}{1-\beta}} k_0^{\frac{\alpha-\beta}{1-\beta}}. \tag{11}$$

Fig. 1 represents that result. Eq. (11) is the upper bound of the set of all reachable goals $\mathcal{S}(k_0, x_0)$ and represents the necessary trade-offs between sustainability objectives.

$\mathcal{S}(k_0, x_0)$ is the set of all achievable sustainability objectives. Any inner pair $(\underline{c}, \underline{x})$ such that $\underline{c} \leq c^+(\underline{x})$ can be guaranteed.[7] Note that on the border, a rise of resource preservation implies a fall of sustainable consumption, meaning the the two sustainability objectives are conflicting.

Static optimization problem on preferences

Using Result (11), the problem (10) is equivalent to the maximization of the

[7] For the sake of simplicity, we will omit the initial state in the notation of function $c^+(k_0, x_0, \underline{x})$.

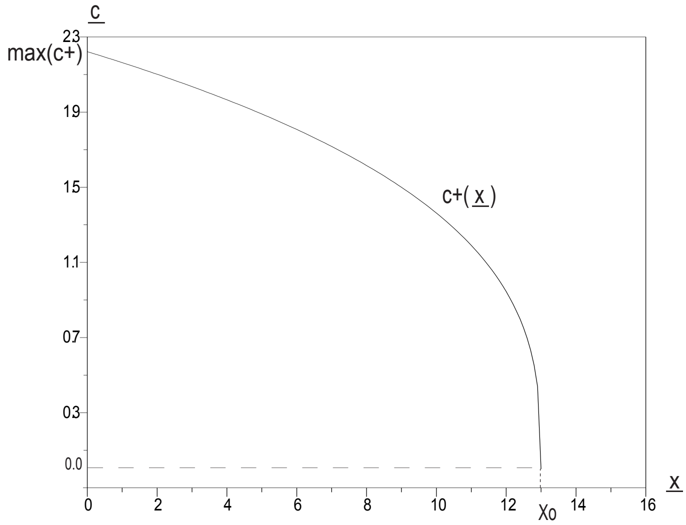


Figure 1: Substitution between guaranteed consumption $\underline{c}$ and resource conservation $\underline{x}$.

criterion among the possible pairs $(\underline{c}, \underline{x}) \in \mathcal{S}(k_0, x_0)$, i.e.,

$$
\begin{aligned}
& \max_{\underline{c},\underline{x}} \mathcal{P}(\underline{c}, \underline{x}) \\
s.t. \quad & \\
& \underline{x} \geq 0 \\
& \underline{c} \geq 0 \\
& x_0 - \underline{x} \geq 0 \\
& c^+(\underline{x}) - \underline{c} \geq 0
\end{aligned}
\tag{12}
$$

This problem is a classical static optimization problem under inequality constraints (Léonard and Long, 1992). Mathematical details of the resolution are given in the appendix.

Depending on the form of the preference function $\mathcal{P}(\underline{c}, \underline{x})$ either inner or corner solutions can occur.

Inner solutions will be characterized by the condition[8]

$$
\frac{dc^+(\underline{x})}{d\underline{x}} = -\left(\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}}}\right)_{|\underline{c}=c^+(\underline{x})} \tag{13}
$$

Fig. 2(a) illustrates this result.

[8]To simplify mathematical formulas, we omit the initial state in the function $c^+(k_0, x_0, \underline{x})$ and only use $c^+(\underline{x})$. We also do not use special marks for optimal values of $\underline{c}$ and $\underline{x}$.

A corner solution with $\underline{x} = 0$ occurs when

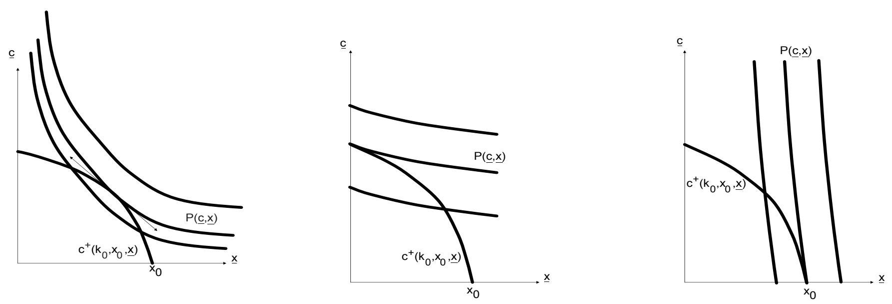
$$0 \leq \left(\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}}}\right)_{|\underline{x}=0;\ \underline{c}=c^{+}(0)} \leq -\left(\frac{dc^{+}(\underline{x})}{d\underline{x}}\right)_{|\underline{x}=0} \tag{14}$$

It means that such a corner solution is possible if the slope of the preference function in $\underline{x} = 0$ is smaller than that of the function $c^{+}(\underline{x})$. Such a corner solution is then possible only if the marginal benefit of preservation for a nil resource stock is small with respect to the marginal benefit of an extra unit of guaranteed consumption. Obviously, it requires that $\mathcal{P}_{\underline{x}|(\underline{x}=0)} < \infty$. Fig. 2(b) illustrates this result.

A corner solution with $\underline{c}^{*} = 0$ occurs when

$$0 \leq -\left(\frac{dc^{+}(\underline{x})}{d\underline{x}}\right)_{|\underline{x}=x_0} \leq \left(\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}}}\right)_{|\underline{x}=x_0;\ \underline{c}=0} \tag{15}$$

The slope of the preference function must be greater than that of the function $c^{+}(\underline{x})$ in $\underline{x} = x_0$. In particular, the marginal preference for guaranteed consumption, when consumption is zero, must be finite (and small with respect to the marginal preference of preservation). Fig. 2(c) illustrates this result.



(a) Inner solutions $\underline{c} > 0$ and $\underline{x} > 0$

(b) Corner solution $\underline{x} = 0$ and $\underline{c} = c^{+}(k_0, x_0, 0)$

(c) Corner solution $\underline{c} = 0$ and $\underline{x} = x_0$

Figure 2: Optimal sustainability objectives

Interpretation of the results for the DHS model

The application of the proposed approach to this canonical model allows us to emphasize its relationship with the maximin approach as it is presented in Solow (1974) and Cairns and Long (2006). Whatever the case, the optimal solution always satisfies $\underline{c} = c^{+}(k_0, x_0, \underline{x})$. The only intertemporal path that maximizes minimal rights is thus a maximin under constraints, and is efficient from an economic point of view (no resource is wasted). There are two ways to take into account environmental and natural resources concerns in the maximin approach.

One can include them as arguments of the utility function, or one can define constraints to restrict the maximin path. But in that second case, the level of the constraint has to be defined. Our approach results in the definition of optimal level of such constraints.

The level of the resource preservation depends on the marginal preferences on guaranteed consumption $\mathcal{P}_{\underline{c}}$ and resource stock preservation $\mathcal{P}_{\underline{x}}$. If we assume that the consumption marginal preference for a nil consumption $\mathcal{P}_{c|(c=0)}$ is infinite, and that that of the resource for a nil resource stock $\mathcal{P}_{x|(x=0)}$ is also infinite, the result must satisfy $\underline{c} > 0$ and $\underline{x} > 0$, and sustainability objectives will be to sustain a positive consumption and to preserve a positive part of the stock natural resource stock.

Last, when the usual criteria do not have solutions, a possibility is to reduce the set of admissible paths (by selecting paths that satisfy a set of constraints) and then to apply a criterion. For example, the green golden rule criterion (Beltratti et al., 1995) has no solution in the canonical model presented in this section. Our criterion is an alternative approach that guarantees that the long run utility is greater than $U(\underline{c}, \underline{x})$.

4 A practical step toward sustainability?

In this section, the relevance of our approach to meet sustainability challenges is discussed point-by-point. Its links with usual approaches, mainly the maximin one, are emphasized, and we discuss its applicability.

Sustainability objectives represented by intertemporal constraints on indicators

The leading idea in our approach to thinking sustainability is that sustainability objectives can be represented by a set of constraints on indicators and their associated thresholds (subsection 2.1). This idea arises from the fact that economic stocks and decisions are not a matter of sustainability in and of themselves, but contribute to the various sustainability issues. Each sustainability issue can be associated with a measurement, namely, a sustainability indicator. In our framework, sustainability indicators are *characteristics* that can be compared to Lancaster's consumption *characteristics* (Lancaster, 1966). A given stock or decision may participate in various sustainability characteristics, each one measured by an indicator. Moreover, the effect of a stock or decision can either be positive or negative on one or several sustainability characteristics, inducing potential conflicts between sustainability issues. Economic decisions for sustainability must then be defined with respect to their consequences on the various sustainability indicators.

Given a set of measurement tools, i.e., the sustainability indicators, sustainability objectives must be defined. In our approach, thresholds are used to distinguish sustainable and unsustainable situations. This requires interpretation. Referring to Friedman and Savage (1948) who proposed to use a concave-convex utility function to explain observed choices, utilities from sustainability *characteristics* could be represented with a convex-concave function, which can be interpreted as follows:

- For high levels of the associated sustainability indicator, there is a decreasing marginal utility, that can even lead to a bliss utility (upper utility boundary) with respect to that sustainability *characteristics*.
- For low levels of the sustainability indicator, utility is very low, with increasing marginal utility.

A limit case of such a convex-concave utility function, with a discontinuity of the function around some threshold, gives us an interpretation of the use of thresholds on indicators.[9] Fig. 3 represents such a convex-concave utility function, and the limit case of an indicator with a threshold.

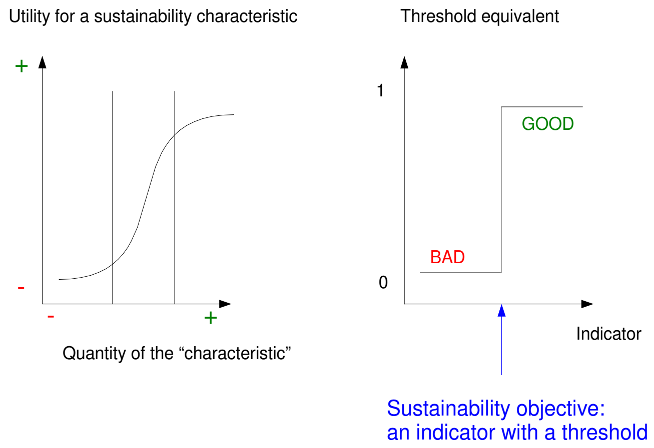


Figure 3: Convex-concave utility and the "Indicator-Threshold" approach.

Based on indicators and thresholds, our approach is consistent with what is done in practice. Targets on indicators can be interpreted as constraints to meet. A theoretical question thus arises. Could the use of indicators and thresholds to represent sustainability objectives be consistent with the given concept of sustainability? In other words, is it possible to conceive a framework based on indicators and thresholds, within which conflicting sustainability issues, including environmental ones, and intergenerational equity are both accounted for?

In our approach, according to definition 3, an economic trajectory is sustainable if and only if all of the constraints representing the sustainability objectives

[9]Strong sustainability constraints may be associated to increasing concave utilities, with a negative infinite limit when the indicator is decreasing toward some lower range value. However, such a case can still be represented by a constraint. It may for example imply a conservation constraint of some natural resource stock to avoid resource extinction.

are satisfied at all times. This means that all of the indicators must be above the objective thresholds, and therefore that all the sustainability issues, including environmental ones, are accounted for in the same way. There are no trade-offs between sustainability objectives. Moreover, in definition 3, all generations are treated with *anonymity*. There are neither discounting nor trade-offs between generations, which are accounted for in the same way. In fact, such sustainability constraints can be interpreted as minimal rights to be guaranteed to all generations. Our approach of sustainability therefore rationalizes the practice of using indicators to deal with sustainability, in terms of the given challenges.

Several more questions arise in turn. How should we define sustainability objectives when they are represented by a set of constraints on indicators that have to be satisfied at all times? If a threshold is a limit case of some concave-convex utility function, how should be set such thresholds? If all sustainability objectives have to be satisfied at all times, how shall we take into account conflicts between objectives when setting those thresholds?

Maximizing minimal rights for sustainability

To define sustainability objectives, the proposed approach consists in maximizing a preference function $\mathcal{P}$ depending on those objectives, i.e., on the constraints thresholds (subsection 2.2). From a general point of view, preference does not necessarily need to be of a cardinal form to apply our approach. An ordinal preference relationship is sufficient in our case. However, assuming a cardinal preference function makes it possible to compare the approach with classical ones, especially other optimization approaches for sustainability, and associated criteria.

Choices among a set of objectives in terms of maximization of a preference function entails assuming that decision makers are supposed to choose as if they had attributed some common quantity to the various objectives, and then selected a combination of objectives that yielded the largest total amount of that quantity. From this point of view, strong sustainability constraints (for example, associated with some critical value for a natural resource asset) can be taken into account in the preference function $\mathcal{P}$ (for example with an infinite marginal preference for increasing the associated indicator at such a critical level), and veto thresholds can be used to indicate situations in which a sustainability objectives is so important at some point that it will require the decision maker to negate any give and takes with other objectives beyond that point (Nowak, 2004).[10] It results in indifference relationships in the space of the sustainability objectives, that can take various shapes:

- Smooth relationships between objectives in preference can represent possible trade-offs;

[10]From a mathematical point of view, it would result in an infinite marginal preference for this objective at that threshold level w.r.t. preferences on other objectives.

- Asymmetric shape, with a discontinuity in a critical threshold value, can represent strong sustainability requirement;
- Leontief type functions can represent complementary cases, when all objectives have to be improved together.

These different cases are illustrated in Fig. 4.

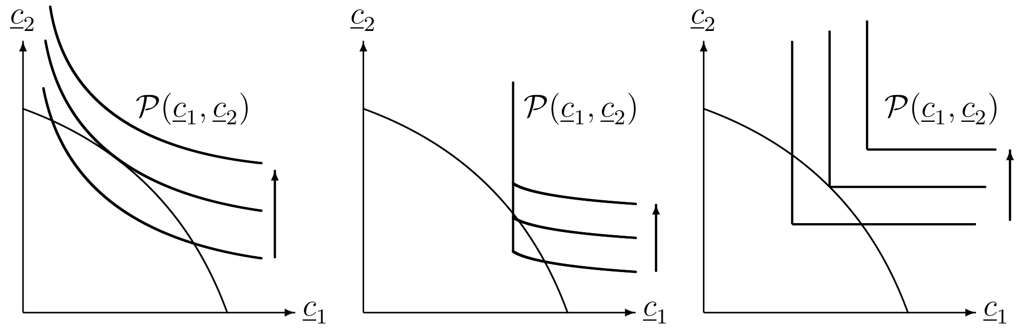


Figure 4: Smooth, constrained, and Leontief Preference functions on two sustainability objectives $(\underline{c}_1, \underline{c}_2)$.

$\mathcal{P}$ is not a welfare function as it is not a measure of some actual utility. It is a particular preference function, based on objectives levels, that ranks combinations of them, and defines trade-offs when they conflict. In some sense, it is close to the Rawlsian approach since it can be interpreted as a way to think of sustainability focusing on minimal rights that all generations would have.[11] Consequently, contrary to usual approaches that result in an intertemporal trajectory interpreted as sustainable because it maximizes a so-called "sustainability" criterion, the present approach does not aim at defining a particular path but sustainability objectives to be achieved by all generations. What is sustained (and optimized) in our model

[11] According to John Rawls' conception of justice (Rawls, 1971), the first requirement for equity is to choose the allocation of resources that provides the maximal number of minimal rights everyone can enjoy. This result comes from the allocation of rights one would made under the "veil of ignorance." Rawls argues that justice should be based on two principles, with a priority order. The first principle is the definition of fundamental rights every one can enjoy ("*each person is to have an equal right to the most extensive scheme of equal basic liberties compatible with a similar scheme of liberties for others*"). The second principle is based on (with here again a priority order) "fair equality of opportunity" to a social position and on the "difference principle" that stipulates that inequality in the wealth distribution is justified if it is beneficial for the poorest individual, i.e., if the poorest individual in this configuration in richer than the poorest individual in all other possible allocations. This last statement leads to the maximin criterion, which is thus the less important point in Rawls' theory of justice. Recent empirical studies show that inequality aversion and maximin concerns may explain some observed choices in interindividual distribution problems (Engelmann and Strobel, 2004). It should be interesting to assess if intergenerational considerations result in the same kind of behavior, which would be an empirical basis to extend the Rawls theory of justice to intergenerational and sustainability issues.

is thus not a utility level, but a set of sustainability objectives. From this perspective, sustainability objectives become the sustainability values (Cairns, 2008). We thus adopts a *deontological* perspective to think of sustainability, when considering actual levels of utility would adopt a *consequentialist* one. In particular, the maximin approach aims at sustaining a utility level, considering only one sustainability objective in the sense of definition 3. If the utility depends on the actual level of the sustainability indicators, i.e., $U(\mathcal{C}_1(X,u),\dots,\mathcal{C}_i(X,u),\dots,\mathcal{C}_I(X,u))$, sustaining utility would require ensuring permanent compensation between indicators levels when they vary.[12]

Considering several sustainability objectives represented by constraints has the great advantage of being easier to implement, as

> *strict conservationist policies that impose explicit exploitation constraints ensure sustainability, and are far simpler to implement, compared to the more complex resource management rules that aim at a careful balancing of costs and benefits.* (Gerlagh and Keyser, 2003, p.312)

Methodology

Further interpretations can be drawn from the methodological part of our framework, described in subsection 2.3. There is an underlying interpretation to each of the two steps of the approach.

By defining the set of achievable sustainability objectives, the necessary trade-offs between these objectives are revealed, as represented in Fig.1 in our illustrative case. This first step implies no normative choice. It leads to the definition of the possibility frontier of our sustainability problem, which can be interpreted as the maximal level of the sustainability characteristics that can be sustained forever.

The second step is a choice on that frontier, depending on preferences. This static optimization is based on a social preference function, representing the preferential trade-offs on several sustainability objectives. The definition of such preferences depends on a normative choice.

We now turn to an important issue: How can our approach be applied to real sustainability issues?

Defining achievable sustainability objectives in a complex economy

Real sustainability issues are characterized by complex dynamics, and uncer-

[12]Note that once the sustainability objectives are defined, in our approach, one only knows that along a sustainable path the actual utility will be greater than the utility associated to the minimal rights:

$$\begin{array}{lll} & \forall i, \forall t, & \mathcal{C}_i(X_t,u_t) \geq \underline{c}_i \\ \Rightarrow & \forall t, & U(\mathcal{C}_1(X_t,u_t),\dots,\mathcal{C}_i(X_t,u_t),\dots,\mathcal{C}_I(X_t,u_t)) \geq U(\underline{c}_1,\dots,\underline{c}_i,\dots,\underline{c}_I) \end{array}$$

tainty. Even if the static optimization problem resulting from the proposed approach is simpler to deal with than a complex dynamic one, the use of our criterion relies on the definition of the set of all reachable objectives. All the difficulty of addressing the dynamic aspect of the problem is in the first step of the analysis, which make the definition of achievable objectives difficult.

The use of targets in problems involving multiple criteria is increasing (Wallenius et al., 2008). In such approaches, decision maker's utility or value function may not depend on the levels of performance on different criteria, but instead on whether the levels meet a target or threshold. The technical tools developed in the operational research literature on Multi-Criteria Decision Making, such as goal programming, could be extended to take into account intertemporal considerations, in order to compute our set of achievable sustainability objectives. The viability approach is a relevant framework to extend such problems to intertemporal dimensions, and to argue the possibility of applying our approach, we can refer to the increasing number of studies that use the viability approach in complex bioeconomic systems, emphasizing its applicability if one accepts the use of numerical methods to solve the problem (for recent examples, see Cury et al. (2005) for ecosystem-based fishery management, and Tichit et al. (2007) or Baumgärtner and Quaas (2009) for agri-environmental issues). In particular, the proposed approach can be extended to take into account uncertainty (De Lara and Doyen, 2008), and it is possible to define the probability that a given set of objectives $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$ is achieved from the initial economic state X_0, and given a set Ω of uncertain scenarios $\omega(.)$, as follows.

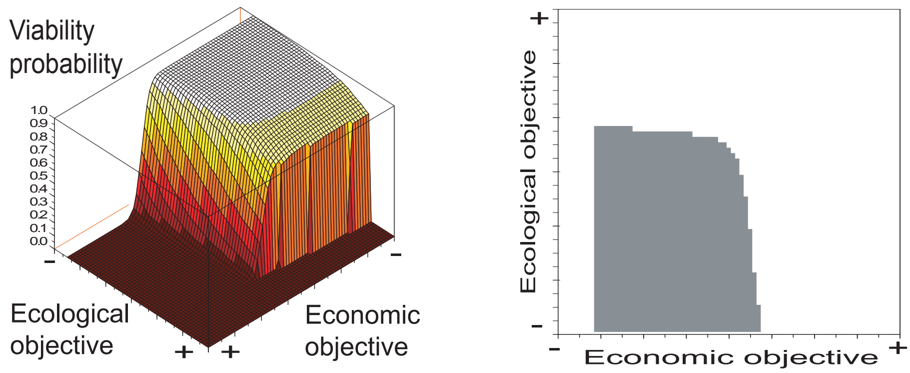
$$\mathbf{P}(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I) = Prob_{\omega(.)} \begin{pmatrix} \exists u(.) \text{ such that, from } X_0, \\ \text{given } \dot{X} = F(X, u), \\ \forall i \in \{1, I\}, \forall t \in [0, T] \\ \mathcal{C}_i(X(t), u(t)) \geq \underline{c}_i \end{pmatrix} \tag{16}$$

Computing that probability on the whole mapping $\mathbb{R}^I \mapsto \mathbb{R}$, it is possible to define the set of objectives that are achievable with a probability greater than any given risk threshold. Such an approach is, for example, applied in De Lara and Martinet (2009), where two sustainability objectives are considered in a multi-fleet and multi-species fishery: an economic objective and an ecological objective. It leads to the kind of results represented in Fig. 5.

Revealing preferences in the real world

The preference function $\mathcal{P}$ is a simple static preference order. By assuming that the sustainability objectives have to be achieved for all generations, our approach results in a static optimization problem, once the set of achievable objectives is defined. It is simpler to implement than standard approaches based on optimal control problems with multi-attribute utility functions. In particular, we avoid the difficulty of defining equitable criteria or preferences on infinite utility streams pointed out in the introduction.

Moreover, if the definition of sustainability objectives is delegated to some policy maker, it is simpler to assess a static preference function like $\mathcal{P}$ than a



(a) Viability probability w.r.t. two sustainability objectives

(b) Objectives achievable with a probability greater than 0.9

Figure 5: Set of achievable sustainability objectives under uncertainty

complex multi-attribute intertemporal welfare function. In particular, it is possible to determine preferences using graphical representations of the achievable goals (Choi et al., 2007). For example, Figs. 1 and 5 can be used as graphical tools to help decision-making in terms of the given problems.

5 Conclusion

Sustainability involves the conservation of "something", to be defined, for future generations. Two of the challenges in defining sustainability objectives are to deal with conflicting objectives, and to ensure intergenerational equity. On the one hand, the way these challenges are addressed in economics results in normative sustainability criteria often hard to apply in practice. On the other hand, sustainability is tackled in practice using indicators, and objectives are defined as thresholds that act as constraints on the indicators. However, the definition of a specific objective rarely takes into account potential conflicts with other objectives and the difficulty of achieving all of them over time.

In this paper, we laid out an approach that defines sustainability objectives, represented by constraints on indicators and their associated thresholds. We assume that sustainability requires that all these constraints are respected at all times. This means that all the sustainability objectives are achieved, including environmental ones, and from an intergenerational equity perspective. This approach is based on a criterion that defines minimal rights to be guaranteed to all generations. Applying this criterion is a two step process. Firstly, the set of objectives that are achievable for all generations, given the economic endowments, is defined, revealing the necessary trade-offs between them. Secondly, preferred sustainability objectives are defined among the intertemporally achievable ones. Such an approach rationalizes the practice of using indicators to deal with sustainability, as the defined sustainability objectives are actually achievable in the long run, taking into account conflicting objectives and intertemporal issues. Moreover, its applicability makes it a practical step toward sustainability.

Our approach is somewhere between weak sustainability and strong sustainability: trade-offs may occur in the preferences between sustainability objectives, but they result in the definition of thresholds for sustainability indicators that can be interpreted as conservationist objectives. According to Gerlagh and Keyser (2003), conservationist policies can be Pareto efficient, and strict resource conservation is equivalent to non-dictatorship of the present, as defined by Chichilnisky (1996). This reinforces the intergenerational equity of our approach.

If, as argued by Howarth (2007), the present society holds a moral obligation to pass a sustainable world to future generations, it may be necessary to impose strict sustainability objectives, for example, the conservation of some specific natural resources. Defining sustainability objectives as constraints on indicators may thus require some commitment to satisfy them in the long run, dealing with the temptation to revise the objectives when economic state evolves (Amador et al., 2006). Such a perspective emphasizes the relevance of the proposed approach as it defines the sustainability objectives in a rational way, avoiding the election of unreachable ones that would be inevitably revised. It also opens a wide range of research opportunities. For example, it would be interesting to study the evolution of the set of reachable objectives along a trajectory. More specifically, an evolution in the size of this set could be a measurement of the consequences of present economic decisions on future sustainability, and the opportunity handed down to future generations.

A Appendix

A.1 Sustainability issue and the viability approach

If thresholds $\underline{c}_i, (i = 1, \ldots, I)$, are known, the problem of defining sustainable trajectories is a viability problem (Aubin, 1991), which purpose is to study the consistency between a dynamic system and so-called viability constraints. From that point of view, in definition 3, sustainability objectives are viability constraints to be satisfied by the economy at all times. The aim of a viability problem is to define if there are intertemporal paths starting from the initial economic state X_0 that satisfy all of the viability constraints forever. In that sense, viability does not allow trade-offs, neither between constraints (all constraints must be satisfied for the system to be said viable), nor between time periods (the constraints must be satisfied at any time for the system to be said viable). In our sustainability framework, it means that all the objectives and all the generations are treated symmetrically, and that if one (or more) of the objectives is not satisfied at some time, the economy is not sustainable. When a viability constraint is not satisfied, the system is said to face a crisis.

In other words, viability focuses on crisis, and aims at defining decisions to avoid them. By introducing the function $\mathbf{1}(\mathcal{C}, \underline{c})$ that is equal to one if the condition $\mathcal{C} \geq \underline{c}$ holds and zero otherwise, one can define the instantaneous viability value function $v(t) = \prod_{i=1}^{I} \mathbf{1}(\mathcal{C}_i(X(t), u(t)), \underline{c}_i)$ that is equal to 1 if all the constraints are satisfied at time t and 0 otherwise. At any instant during which one of the constraint is not respected, the system is said to face a crisis. In such a case, $(1 - v(t))$ is equal to 1. A viability problem consists in defining paths that do not face a crisis. It can be stated as the following minimization problem

$$\begin{aligned} V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X_0) &= \min_{u(.)} \int_0^{+\infty} (1 - v(s))ds \\ s.t.\ & \dot{X} = F(X, u) \\ & X(0) = X_0 \end{aligned} \tag{17}$$

Since $1 - v(s)$ is only lower semicontinuous, the usual tools of optimal control in an infinite horizon do not generally apply. The purpose of the viability theory is to solve such problems.

If $V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X_0) = 0$, there exist decisions $u(.)$ generating trajectories $X(.)$ starting from X_0 that satisfies all the viability constraints at all times. It means that the given sustainability objectives $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$ are achievable from the initial state X_0.

If $V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X_0) > 0$, there are no intertemporal decisions $u(.)$ resulting in economic trajectories that satisfies all the viability constraints at all times. It means that, given the economic endowments, the given sustainability objectives $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$ can not be achieved for all generations and that some of the constraints will be violated for some of the generations. In such a situation, there are two options.

On one hand, the system can be driven toward a new economic state $\tilde{X}$ such that $V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(\tilde{X}) = 0$, if such states exist.[13] It means that before reaching such a state, some of the constraints will be violated.

On the other hand, one can modify the viability constraint levels and choose new sustainability objectives $(\tilde{c}_1, \ldots, \tilde{c}_i, \ldots, \tilde{c}_I)$ such that $V_{(\tilde{c}_1,\ldots,\tilde{c}_i,\ldots,\tilde{c}_I)}(X_0) = 0$. One then faces the problem of defining sustainability objectives.

The first option would imply that some of the sustainability objectives are not achieve for some generations. As we are concerned with the intergenerational equity issue, we want to define minimal rights to be guaranteed to all generations. We thus ruled out, in the present paper, the sacrifice of some generations to improve the sustainability of the economy, and discussed only the second option. Nevertheless, a quite straightforward extension of the viability problem makes it possible to address that recovery issue. We just provide here the intuition to the reader: If it is not possible to define another set of sustainability objectives (for

[13]The set of states X such that $V_{(\underline{c}_1,\ldots,\underline{c}_i,\ldots,\underline{c}_I)}(X) = 0$ is called the viability kernel of the problem (Aubin, 1991). It is the set of states from which sustainable economic trajectories (as defined by definition 3) start. If that set is empty, there are no economic states that allow to satisfy the constraints in the long run, and this first option is thus not possible.

example because thresholds levels correspond to some basic needs and can not be reduced), it is necessary to improve the sustainability of the economy such that the objectives will be achievable in some future. In the sufficientarianism framework, Chichilnisky (1977) aimed at defining how to reach economic development paths that satisfy basic needs, arguing that economic development must be consistent with the attainment of adequate levels of *per capita* consumption of basic goods. She proposed a criterion minimizing the time needed to reach an economic path that satisfies the basic needs, defining efficiency with respect to the minimization of the time horizon after which they are satisfied. The present approach offers a quite similar, and straightforward, way to define how to reach higher sustainability objectives in the future, when economic endowments only make low sustainability objectives achievable. Such an extension is based on the concept of "minimum time of crisis" (Doyen and Saint-Pierre, 1997) and comes directly from the interpretation of the viability criterion (17). This criterion minimizes the cumulated duration of periods during which some of the viability constraints are not satisfied, which in some sense can be interpreted as the number of generations that do not benefit from the minimal rights represented by the sustainability objectives. This "time of crisis" is positive if the objectives $(\underline{c}_1, \ldots, \underline{c}_i, \ldots, \underline{c}_I)$ are not achievable with economic endowments X_0. This approach however differs from sufficientarianism in an important way for our sustainability issue: in the minimum time of crisis approach, nothing requires that the first generations are the ones that do not benefit for sustainability minimal rights, as we do not minimize a time horizon (which would imply to make the first generations do sacrifices to improve the sustainability of the economy) but the minimal number of generations (anywhere in time, it depends on the optimal solution) that will face an unsustainable situation. There is thus some kind of anonymity as we just minimize the number of generations facing unsustainability, without references to their place along time.

A.2 Computation of optimal sustainability objectives in the DHS model

To solve the static optimization problem defined by Eq.(12), we define the following functional form

$$\phi(\mu_1, \mu_2, \mu_3, \mu_4, \underline{c}, \underline{x}) = \mathcal{P}(\underline{c}, \underline{x}) + \mu_1 \underline{x} + \mu_2 \underline{c} + \mu_3 (x_0 - \underline{x}) + \mu_4 (c^+(\underline{x}) - \underline{c}) \quad (18)$$

where the μ_j, $j = 1, \ldots, 4$, are the dual variables of the problem.

According to the Khun-Tucker theorem, the optimality conditions of the problem are[14]

$$\begin{aligned}
\phi_{\mu_1} &= \underline{x} \geq 0, & \mu_1 &\geq 0, & \mu_1\underline{x} &= 0 & (19)\\
\phi_{\mu_2} &= \underline{c} \geq 0, & \mu_2 &\geq 0, & \mu_2\underline{c} &= 0 & (20)\\
\phi_{\mu_3} &= x_0 - \underline{x} \geq 0, & \mu_3 &\geq 0, & \mu_3(x_0 - \underline{x}) &= 0 & (21)\\
\phi_{\mu_4} &= c^+(\underline{x}) - \underline{c} \geq 0, & \mu_4 &\geq 0, & \mu_4(c^+(\underline{x}) - \underline{c}) &= 0 & (22)\\
\phi_{\underline{x}} = \mathcal{P}_{\underline{x}} + \mu_1 + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}} &\leq 0, & \underline{x} &\geq 0, & \underline{x}\left(\mathcal{P}_{\underline{x}} + \mu_1 + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}}\right) &= 0 & (23)\\
\phi_{\underline{c}} = \mathcal{P}_{\underline{c}} + \mu_2 - \mu_4 &\leq 0, & \underline{c} &\geq 0, & \underline{c}(\mathcal{P}_{\underline{c}} + \mu_2 - \mu_4) &= 0 & (24)
\end{aligned}$$

Strictly positive solutions

First assume that, at the optimum, both the optimization variables $\underline{x}$ and $\underline{c}$ are strictly positive. From eq. (19) and (20), we get $\mu_1 = \mu_2 = 0$. Moreover, if consumption is positive, the preserved resource stock will be lower than the initial stock x_0. Eq. (21) then leads to $\mu_3 = 0$. We thus get a system from equations (22), (23) and (24), in which there are three equations and three variables.

$$\mu_4(c^+(\underline{x}) - \underline{c}) = 0 \quad (25)$$

$$\underline{x}\left(\mathcal{P}_{\underline{x}} + \mu_4 \frac{dc^+(\underline{x})}{d\underline{x}}\right) = 0 \quad (26)$$

$$\underline{c}(\mathcal{P}_{\underline{c}} - \mu_4) = 0 \quad (27)$$

As we have assumed that $\underline{c} \neq 0$, eq. (27) leads to $\mu_4 = \mathcal{P}_{\underline{c}} > 0$. We thus get from eq. (25) and (26) the conditions

$$\underline{c} = c^+(\underline{x}) \quad (28)$$

$$\frac{dc^+(\underline{x})}{d\underline{x}} = -\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}}} \quad (29)$$

It leads to the following result

$$\frac{dc^+(\underline{x})}{d\underline{x}} = -\left(\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}}}\right)_{|\underline{c}=c^+(\underline{x})}$$

Corner solution $\underline{x} = 0$

Assume now that $\underline{x} = 0$. It implies that $\mu_3 = 0$ (from eq. 21).

If $\underline{c} = 0$, eq. (22) would require $\mu_4 = 0$. But it is in contradiction with relation (24) which requires $\mathcal{P}_{\underline{c}} + \mu_2 - \mu_4 \leq 0$. Thus, we have $\mu_4 > 0$ and $\underline{c} = c^+(0)$, from eq. (22).

[14] The variables are at optimal values. In order to simplify notation, we do not denote them by $\underline{x}^*$ and $\underline{c}^*$ but simply by $\underline{x}$ and $\underline{c}$

As $\underline{c} \neq 0$, we get $\mu_2 = 0$ from eq. (20). We then get $\mu_4 = U_{\underline{c}}$ from eq.(24). Finally, the inequality condition (23) requires

$$\mathcal{P}_{\underline{x}|(\underline{x}=0)} + \mu_1 + \mathcal{P}_{\underline{c}|(\underline{c}=c^+(0))} \left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|(\underline{x}=0)} \leq 0 \tag{30}$$

This equation can be expressed with respect to μ_1

$$\mu_1 \leq -\mathcal{P}_{\underline{x}|(\underline{x}=0)} - \mathcal{P}_{\underline{c}|(\underline{c}=c^+(0))} \left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|(\underline{x}=0)} \tag{31}$$

As $\mu_1 \geq 0$, it is possible only if $-\mathcal{P}_{\underline{x}|(\underline{x}=0)} - \mathcal{P}_{\underline{c}|(\underline{c}=c^+(0))} \left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|(\underline{x}=0)} \geq 0$, or equivalently if

$$0 \leq \left(\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}}}\right)_{|\underline{c}=c^+(0);\ \underline{x}=0} \leq -\left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|(\underline{x}=0)} \tag{32}$$

Corner solution $\underline{c} = 0$

We now turn toward the other case : $\underline{c} = 0$. The inequality from eq. (24) implies

$$\mathcal{P}_{\underline{c}|(\underline{c}=0)} + \mu_2 - \mu_4 \leq 0 \quad \Longrightarrow \quad \mu_4 \geq \mathcal{P}_{\underline{c}|(\underline{c}=0)} + \mu_2 > 0 \tag{33}$$

It is only possible if $\mathcal{P}_{\underline{c}|(\underline{c}=0)} < \infty$.

As $\mu_4 > 0$, we know from eq. (22) that $\underline{c} = c^+(\underline{x})$ which means, as $\underline{c} = 0$, that $\underline{x} = x_0$. We have $\mu_1 = 0$ from eq. (19). Thus, $\underline{x} > 0$ requires from eq. (23) that

$$\mu_4 = -\frac{\mathcal{P}_{\underline{x}|(\underline{x}=x_0)}}{\left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|\underline{x}=x_0}} \tag{34}$$

Combining this condition with eq. (33), we get

$$0 \leq \mu_2 \leq -\mathcal{P}_{\underline{c}|(\underline{c}=0)} - \frac{\mathcal{P}_{\underline{x}|(\underline{x}=x_0)}}{\left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|\underline{x}=x_0}} \tag{35}$$

We thus have a condition on the marginal preferences in $(\underline{c} = 0, \underline{x} = x_0)$:

$$0 \leq -\left(\frac{dc^+(\underline{x})}{d\underline{x}}\right)_{|(\underline{x}=x_0)} \leq \left(\frac{\mathcal{P}_{\underline{x}}}{\mathcal{P}_{\underline{c}}}\right)_{|\underline{c}=0;\ \underline{x}=x_0}$$

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