Making Greenhouse Gas Policy Decisions under Uncertainty: A Computable General Equilibrium Approach[1]

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Abstract: Large-scale greenhouse gas (GHG) abatement policies are subject to a range of uncertainties that may significantly affect their outcomes, including emissions reductions achieved in practice and their associated costs. Economic models, including those of the computable general equilibrium (CGE) variety, have become important tools for developing and analyzing these policies. Such models, however, are with few exceptions purely deterministic, and by virtue of their increasing detail and complexity have severe limitations for forms of uncertainty analysis that could provide policy-makers with valuable information on the risks as well as the potential benefits of GHG abatement, and how to hedge against these risks. This paper describes a pragmatic method for introducing and analyzing key uncertainties with a heretofore deterministic CGE model that is being used by the State of California to implement its aggressive GHG reduction goals. The method allows for existing information, including the model itself as well as its input data, to be incorporated into uncertainty analysis, and yields policy-relevant and accessible insights into the potential effects of uncertainty on projected GHG policies and how they might be addressed by policy-makers.

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1. Introduction

Designing large-scale greenhouse gas (GHG) reduction policies presents decision-makers with a range of formidable analytical challenges. Technical data from disparate sources must be assembled and organized, complex inter-relationships within the energy system and the economy accounted for, and the possible effects of policies that may not reach fruition until years into the future estimated. Over the past several decades, quantitative mathematical models of the economy and the energy system, implemented in computer code, have emerged as primary tools for these purposes in analyzing the large-scale economic aspects of multi-sector GHG emissions abatement, and for identifying policy strategies that simultaneously meet environmental and economic objectives.

In California, a model of this type, the "Environmental Dynamic Revenue Analysis Model," or "E-DRAM," is one of the suite of models being used by the Air Resources Board (CARB) to develop and implement the regulatory details of AB32, the Global Warming Solutions Act of 2006 (CARB 2008a,2008b, 2008c). E-DRAM is a "computable general equilibrium" or "CGE" model representing the California economy, and is an extended and enhanced version of a model first developed by the California Department of Finance and the University of California at Berkeley in the 1990s to analyze state budgetary and fiscal policy (Berck et al. 1996). It has previously been used to analyze policies including petroleum independence, reductions of CO2 emissions from vehicles, and alternative fuels policy. The development of the current form of the model, which incorporates a range of inputs and features specifically directed toward GHG policy analysis, was supported in part by PIER (Berck et al. 2008).

Like virtually all numerical energy-economic models presently applied to GHG policy, E-DRAM is deterministic – that is, it does not directly represent the manifold uncertainties attendant to all aspects of large-scale emissions abatement, nor are its outputs formatted or structured in a manner that facilitates addressing these uncertainties. For example, ARB's AB32 "Scoping Plan" is based on a large number of disparate estimates for the costs and outcomes of various policies and measures, which are reflected in E-DRAM as point estimates. Moreover, the model's standard input set also includes point estimates of such exogenous drivers as fuel prices and population and economic growth, uncertainties in which may have substantial but currently unanalyzed effects on modeled policy outcomes.

This modeling philosophy reflects, among other influences, established regulatory and political constraints on GHG policy development. The complexity of analyses using existing deterministic models is already sufficiently great that, all else being equal, adding quantitative uncertainty techniques may exceed the capacity of the policy-making infrastructure to absorb or act on the results. To take a concrete example, like almost all environmental legislation, GHG policies such as AB32 are crafted and prospectively gauged in terms of specific, non-stochastic, quantitative targets. *Prima facie*, even if uncertainty information were readily available, it is by no means apparent how it might be incorporated in a practical and useful fashion into the existing policy process.

At the same time, however, it must be acknowledged that unavoidable uncertainties in multiple dimensions will necessarily affect the implementation of AB32 and its energy, emissions, and economic consequences. At least some of the plan's policy measures will perform differently than projected, and among these it is highly likely that some will deviate significantly from expectations, whether they exceed them or fall short. As events of 2008-2009 forcefully demonstrate, unforeseen developments in the national or world economy may affect California in ways that bear heavily on the economic conditions determining the policy's costs and benefits. The same consideration applies to volatility in energy markets.

It is important to emphasize that, in the case of AB32 as well as more generally, the existence of such uncertainties in no way calls into question either the rationale for GHG abatement policies or the current approaches to designing and implementing them. Rather, it highlights the fact that the GHG policy process is necessarily one of decision-making uncertainty and risk management.

There is thus a dilemma for GHG policy: The concurrence of institutional and other impediments to uncertainty analysis with the presence of uncertainties that are sure to affect the implementation and outcomes of policy. Unfortunately, this state-of-affairs is exacerbated by the fact that existing quantitative methods for energy and environmental decision-making under uncertainty – particularly those involving computer models – are not developed to a degree, or in a form, that might make them practically useful for policy-making.

It is widely acknowledged that better treatment of uncertainties is an important frontier for energy-economic modeling of GHG policy, but progress on this front has been remarkably slow, particularly given that this area of model was initiated more than three decades ago. It is a truism, however, that making already complex models even more complex by the addition of uncertainty information will not necessarily contribute to practical improvements in policy-making. Therefore, in addition to addressing the aforementioned technical hurdles, generating relevant information on important uncertainties must be carried out in a manner that is both tractable and readily useable by policy-makers.

This paper discusses an approach to CGE model-based uncertainty analysis aimed at extending existing analytical methods while providing practical value to policy-makers. While the approach is conceptually more general, our implementation uses the E-DRAM model, and information from the AB32 process, in order to build upon existing capabilities, knowledge, and institutional expertise. The detailed methodology builds on an existing Monte Carlo technique for sensitivity analysis of computable general equilibrium models, "stochastic CGE" analysis, and also draws upon insights from numerical modeling and decision-analysis in both economics and other fields. The goal of this work is to provide California regulators greater flexibility in AB32 design and implementation, insight into the important drivers of the outcomes of policy decisions, and the ability to formulate regulations that are robust against changes in these drivers that cannot be precisely predicted in advance.

The paper is organized as follows. The next section discusses several key themes in the area of energy-economic modeling and uncertainty analysis, introduces our approach at a conceptual level, and summarizes several important practical aspects. The following section is an overview of E-DRAM, including the representation of technological change and policies in the model, and the manner in which it is being used in AB32 analysis. We then turn to the details of our use of the stochastic CGE method, including data sources, error modeling, and the programming done to apply the method to E-DRAM. Initial results and a discussion of policy and decision analysis are then presented. We conclude with remarks on this research direction.

2. Energy-economic modeling and decision-making under uncertainty

2.1. Barriers to uncertainty analysis

On a technical level, the continued prevalence of the deterministic approach in quantitative energy-environmental modeling reflects, among other factors, the challenges - including both cost and complexity – of identifying appropriate sources of information on key uncertainties such as model parameters and inputs, incorporating them into the models, and conducting quantitative uncertainty analysis. These challenges are exacerbated by the development path that this type of modeling has followed since its emergence in the 1970s.

In that decade, computer resources became widely available to researchers, concurrent with the sudden elevation of energy as a paramount public policy focus. These developments jointly facilitated the creation of the first numerical energy models. The computational state-of-the-art at the time, however, compelled modelers to place a premium upon simplicity both in the overall size of the models and in their solution methods. Moreover, model parameterizations were necessarily accomplished primarily *ad hoc*, in that the empirical resources available to generate parameters and provide other model inputs were also scarce.

These origins were key sources of a modeling philosophy that persists today, of informal standards for empirical grounding of models, including not just parameterization practices but also validation and verification. As computational technology improved, it was applied primarily to increasing the size of the models in terms of such structural features as energy sector and technology detail. The resulting demand for larger numbers and types of parameters and other inputs was not generally matched by research to gather and analyze the data needed to generate them. In particular, statistical sampling and estimation methods for this purpose are the exception, with the result that even elementary statistical information such as confidence intervals are commonly unavailable for these quantities. Instead, model inputs are obtained for the most part as a matter of convenience from other sources, and thus take the form of point estimates.[3]

[3] A notable exception is the work of Dale Jorgenson of Harvard University and his collaborators, who over several decades have created and refined an intertemporal computable general equilibrium model of the U.

The confluence of these methodological influences, among others, is a key reason for the current prevalence of very complex deterministic models for energy-environmental applications. Even the narrowest form of uncertainty analysis – introducing probability information for individual parameters and analyzing its effect – can be difficult. Data for estimating probability distributions corresponding to model parameters may be unavailable; this is especially the case for joint distributions of multiple parameters.

Even when these hurdles are overcome, the underlying structure and complexity of many models raises others. Current models are based almost exclusively upon deterministic solution approaches, specifically multi-dimensional equation solving or deterministic mathematical programming methods. For technical reasons, this aspect both reflects the emphasis on increasing model size that we noted above, and facilitates further size increases. Numerical methods for solving models that are mathematically based upon theoretical techniques for decision-making under uncertainty – such as expected utility maximization or expected cost minimization - continue to improve but remain subject to the so-called "curse of dimensionality," i.e., they are in practice numerically intractable except for models with relatively small numbers of variables and equations.

As discussed by Kann and Weyant (2000), there have been several noteworthy examples of energy-economic models applied to decision-making under uncertainty. For the reasons just summarized, however, these have been either low-dimensional models or larger models with a very small number of uncertainties included. What economists consider the "gold standard" for uncertainty modeling – what is known as dynamic stochastic general equilibrium modeling – remains in practice inapplicable to standard, high-dimensional energy-economic models.

2.2. A limitation of standard methodology

Notwithstanding the increasing reliance on numerical models for very detailed and specific quantitative policy analysis, the modeling community's views on how their results should be interpreted and applied to policy may be rather more circumspect. In an authoritative statement, Peace and Weyant (2008) expound the argument that this type of modeling is for "insight, not numbers:"

> "Models are an invaluable tool for exploring alternative policy choices and for generating insights about how the economy might respond to different types and forms of regulation. They cannot, however, predict future events, nor can they produce precise projections of the consequences of specific policy" (Peace and Weyant *op cit*.).

In practice, model-based policy analyses are invariably structured around changes from "baseline," "reference," or "business-as-usual" projections. Thus, a policy such as an emissions tax or an incentive for low-carbon technology will be super-imposed on the

S. economy, emphasizing energy markets, based upon econometric estimation. To our knowledge, this is the only such example in the realm of energy modeling, at least in the U. S.

baseline and the resulting costs, energy consumption, or emissions calculated, and compared to those in the baseline. It is commonly believed that this type of deterministic "incremental" calculation yields useful insights even in the presence of unaccounted-for uncertainty because the results are likely to be more-or-less invariant across a range of possible baseline cases. Indeed, this argument is used to justify the typical focus on single rather than multiple baselines.

Technically, whether or not incremental results obtained in this approach are relatively insensitive across different baselines is an empirical question about the behavior of models – essentially their implicit "second derivatives" with respect to certain inputs. This is a test that has rarely been performed for specific models. Even if the claim is credible, however, the incremental form of analysis as an indirect method of addressing uncertainty encounters a fundamental problem in application to the most common type of current GHG policy, *absolute* emissions targets to be reached at specified future dates. Whether or not the target is met is arguably the primary uncertainty for such policies, and the contributions to this uncertainty of those factors that also drive uncertainty in baselines cannot be addressed by the simplest form of incremental analysis.

To illustrate, consider an economy in which present-day emissions are X tons of CO2, and a target of 50% of X is set to be achieved by some future date D. A baseline projection is that without new policies, emissions at D will be 125% of X, so that a reduction of 60% from the baseline by D is required to meet the target. A model is used to identify a set of policies that will accomplish this goal, and to estimate their costs.

Now, there is assumed to be uncertainty in the baseline arising from such factors as population and economic growth rates, and fluctuations in energy prices. It is claimed that the modeled estimates of incremental emissions reductions and costs will be roughly unchanged across different baselines. Even if this is valid, however, it is not the case that the core outcome of actually meeting the reduction target, or of doing so at a given cost, is also invariant.

The reason is simple: The set of policies needed to reach the emissions reduction target were calibrated from the a priori baseline. Suppose baseline emissions were to instead be 140% of X. Then, assuming rough incremental invariance of the policy effects with respect to the model, the initial policy portfolio would result in a reduction to 65% of X. Conversely, perhaps baseline emissions at D turn out to be 110% of X; then the policies would result in emissions at D lowered to 35% of X. In this case, of course, the target is more-than-met, so the policies could be said to have succeeded. However, if it is also the case that there is a priority on minimizing policy costs – which is most certainly the case in practice – this outcome would have entailed over-expenditure relative to meeting the target that was actually set.

Model-based incremental GHG policy analysis reflects both economic cost-benefit and engineering technology methods. The point of this example is that the common argument that this approach is a means of indirectly addressing uncertainty is not assured to be well-grounded when policy takes the form of an exogenous, absolute target.

2.3. An alternative approach

Given uncertainty that any particular policy portfolio will meet the objective target, how should regulators respond? This appears to be a form of portfolio analysis problem, in which a set of investments with individually uncertain payoffs is chosen so as to optimize an expected outcome. The fundamental insight of portfolio analysis concerns the joint structure of risk and uncertainty among multiple investments of this type, and in particular the importance of "diversification," i.e., that an optimal portfolio provides for hedging of risk. A portfolio analysis approach to meeting a 33% renewable electricity standard was recently examined by the CEC (2008). In GHG applications, this approach might take the form of choosing a portfolio of emissions-reduction measures, each of which has some uncertainty attending its performance, so as to optimize a policy criterion such as the expected emissions reductions achievable at a given cost.

The approach we describe in this paper is similar in spirit to formal portfolio analysis, but technically different and in fact simpler. It can be thought of as an integration of sensitivity and uncertainty analysis for model-based GHG decision-making, and is intended to address the several challenges we have discussed: 1) The need to provide information that enhances the existing policy process; 2) The continued limitations of numerical economic modeling for uncertainty analysis; 3) The quandary arising from the conjunction of incrementally-based policy analysis, intrinsic uncertainty, and policy goals set in terms of an absolute, exogenous criterion.

To further motivate the approach, we return to the example of the previous section. A decision-maker in this situation might follow any of several courses. One is to ignore the uncertainty and use only the basic, deterministic information. Another is to over-correct, that is, to implement a set of policies that with this same information would lead to emissions reductions more stringent that the target. Conversely, the decision-maker might instead implement a weaker set of policies, i.e., that according to the deterministic baseline information would fall short of meeting the target. A reason for this would be the possibility that the resolution of uncertainty by the date that the target must be met might actually result in a favorable outcome compared to the *ex ante* deterministic projection.

What these possibilities illustrate is that, absent further information or additional structure imposed on the problem, there is no single, objectively-optimal rule for the decision-maker to follow. The appropriate course of action depends upon, among other things, the decision-maker's "loss function" or "utility function," which in colloquial terms is the criterion used to weigh the consequences of different possible outcomes that may result given the uncertainty.[4] With regard to the choice of actions in the previous paragraph, for example, whether the decision-maker chooses to over- or under-correct in this example will depend in part upon how she weighs the outcome of missing the target against that of paying a higher cost in order to increase the emissions reduction. In

[4] The term "loss function" is used in the statistical decision theory literature, while "utility function" is more common among economists.

abstract terms, the loss function is the rule according to which this trade-off (possibly along with other factors) is evaluated.

Regardless of the loss function, introducing probabilistic information is generally challenging when a complex simulation model is involved, as we pointed out above. Nevertheless, this strategy is sometimes used along with Monte Carlo techniques as a means of model sensitivity analysis. Indeed, in our illustrative example we have implicitly assumed that some form of this kind of analysis has been conducted, in order to estimate the effects of uncertainty upon the modeled relationships between GHG reduction measures and emissions at the future date. But as the example also illustrates, without further elaboration, such analysis frames the problem but does not, *per se*, yield a decision rule for solving it.

To address this problem we take the perspective of what is known as "inverse analysis," a methodology that, with various technical realizations, appears in a number of scientific disciplines. In modeling terms, conventional "forward" analysis is simply the computation of outputs as a function of inputs. By contrast, inverse analysis focuses on the question, given a specific model output, what inputs might have yielded it? Whereas a (well-functioning) model will always give the same output for a given input, the inverse problem is commonly "ill-posed," that is, in many different modeling domains numerous different inputs are likely to give the same output.

The standard deterministic approach to energy-economic modeling of GHG targets is of course "forward:" A baseline simulation is run, then policies imposed and the model outputs re-computed. By contrast, with a specific target being the output of interest and in the presence of uncertain inputs, an inverse perspective poses the question: What various policy portfolios are required, as a function of the uncertain inputs, to ensure that the target is met?

To illustrate – and begin introducing the specifics of our approach – suppose that the uncertainty is in economic growth, and that the mechanism to achieve the GHG reduction in question is a clean energy standard of the form, *x*% of electricity generation must be zero-carbon. In forward GHG modeling, different economic growth scenarios are fairly common, but they would generally take the form of analyzing the sensitivity of the effects of the standard – say in terms of economic cost - to this uncertain variable. By contrast, with a fixed GHG target, it is quite natural to ask, how should the standard be set – what should *x* be - in order to meet the target in each of the different economic growth scenarios? Conceptually, in this example this question could be answered by a series of model simulations, each based on a different growth scenario, each with a different setting of the standard level. Taking this reasoning a step further, if there is a probability distribution available for the uncertain economic growth rate, then this series of simulations could be driven by random sampling from this distribution. And finally, if there were multiple uncertain input variables, with a joint probability distribution, then the same procedure would apply to sampling from this larger space.

This logic provides a way of structuring model-based Monte Carlo-type sensitivity or uncertainty analysis that organizes the information so that it can be focused on, and will inform, the policy decision. The contrast here is to "forward" Monte Carlo simulations that simply add uncertainty to a modeled outcome. In our example, the presumption is that costs increase along with emissions reduction as the standard increases in stringency. Thus, for different values of the uncertain inputs, costs will vary according to the increased stringency required in the baseline resulting from those particular values. By virtue of the way we have constructed this conceptual experiment, the relevant uncertainty is that, while the choice of the clean energy standard level corresponding to each particular sample value of the uncertain inputs will (according to the model) ensure that the GHG target is met, that choice may not result in the target being met for other values. This risk can be hedged against by making the standard more stringent, but doing so increases the cost. The information resulting from the procedure we have sketched allows the decision maker to gauge the value of different levels of confidence that the GHG target will be met. Suppose, for example, that the decision-maker wants 90% confidence; then the clean energy standard can be set so that it results in emissions being no more than the target with probability 0.9 in the distribution of model outputs.[5] In addition, the cost implications of increasing confidence to, say, 95%, or lowering it to 80%, can also be evaluated. Conversely, the information can be used to determine the likelihood of meeting the target as a function of the cost of the clean energy standard. The decision-maker might start with a preferred cost level, and then determine the likelihood associated with it, and the change in likelihood resulting from a decision to increase or lower the cost.

These "dual" possibilities are equally valid ways of analyzing the problem, depending on the decision-maker, or more precisely the decision-maker's loss function. Neither is guaranteed to result in an "optimal" decision, because there is no objective definition of that concept in the setting we have described. We think of the approach we have discussed here as facilitating a form of *robustness* analysis in the sense of helping decision-makers better understand how policies may be affected by uncertainty and how to craft policies that are likely to yield desirable, if not "optimal," outcomes given this uncertainty.

2.4. Overview of application to E-DRAM and AB32 analysis

Our "test-bed" for implementing these ideas is the E-DRAM model in conjunction with key inputs on GHG reduction measures that have been used in the model's AB32 simulations. We apply the "stochastic CGE" technique of Adelman and Berck (1991), which is a Monte Carlo method for analyzing parameter uncertainty in CGE models and its effects on model outputs. In a more recent application, Beckman and Hertel (2009) review the results from many recent CGE estimates of the costs of GHG restrictions and examine their potential sensitivity to parameter changes. Subjecting their model to a stochastic set of external prices and then recording how model determined results like GDP, they are able to revise specific model specifications to improve empirical validity. Others have used this approach to analyze CGE uncertainty in trade specifications

[5] This discussion assumes certain "well-behavedness" properties of the model being used.

(Welsch, 2008) and the substitutability between capital and energy (Kemfert and Welsch, 2000).

There are numerous uncertainties in the underlying parameters of E-DRAM as well as its exogenous inputs, including both price and socio-demographic variables and inputs representing AB32 measure costs and benefits. For this initial analysis, we focus solely on three exogenous inputs: Oil price, natural gas price, and the rate of state economic growth. While this small number of uncertainties is appropriate for a demonstration analysis, it is also the case that these drivers are known to influence the California economy and energy system in a manner, and to a degree, that may have significant effects on AB32 implementation. Different rates of economic growth, for example, would result in different baseline levels of 2020 energy demand, while sufficiently large changes in these energy prices could be expected to affect consumption of the fuels of a magnitude that may affect the outcomes of certain AB32 CO2-reduction measures to a substantial degree, whether by stimulating or suppressing demand underlying demand. We have considered several approaches to generating a joint probability distribution of the values of these three inputs in 2020 in order to conduct the stochastic (Monte Carlo) analysis. For the present, as described in more detail in the following section, we have adapted error information from forecasts of the U. S. Energy Information Administration.

We draw samples of the vector of these forecasted inputs, and enter these draws successively as parameters in E-DRAM using the version of the model containing the publicly-reported savings and cost estimates of GHG-reduction measures analyzed for the Scoping Plan. 100 such "draws" and simulations are executed. The E-DRAM computer code was augmented for this purpose.

Each of these simulations yields a complete set of EDRAM outputs; we focus on Gross State Product (state personal income), labor employment, and aggregate CO2 emissions. The procedure just described will generate distributions of these quantities, which we will use to analyze their sensitivity, as computed by EDRAM, to the stochastic inputs.

The core AB32 E-DRAM inputs are vectors of cost and benefit estimates for portfolios of GHG reduction measures. In practice, these are combined with a carbon price in order to achieve the overall AB32 target. Thus, there is an added layer of complexity with respect the conceptual discussion in the preceding sub-section: For each of the 100 draws-and-simulations of E-DRAM, we also re-calculate the measure-portfolio and carbon price combination needed to reach the target for the specific values of the uncertain inputs in that draw. We then present and discuss the results in the framework of the "dual" perspective noted previously, and give several examples, emphasizing the similarities and differences from the existing baseline-based scenario of the Scoping Plan.

To describe these steps in detail, we begin with an overview of E-DRAM and its use in the AB32 process.

3. The E-DRAM model and its application to AB32

One of the key achievements of contemporary economics is the comprehensive theoretical analysis of how market economies function and of the conditions under which they yield privately and socially optimal allocations of goods and services. This so-called "general equilibrium" theory is a mathematical formalization of ideas dating to the 19th century and even earlier, particularly the insights of Adam Smith. It is primarily credited to the economists Kenneth Arrow and Gerard Debreu, who were awarded the Nobel Prize in Economic Science for their work.

Over the past five decades, this theoretical work has been combined with computational methods to create "computable general equilibrium" (CGE) models, which are numerical realizations of the abstract theory. CGE models are now widely used in trade and development economics, fiscal studies, and public finance, as well as in energy and environmental analysis.

E-DRAM is a CGE model of the California economy. It represents the overall relationships among California producers, California households, California governments, and the rest of the world. Rather than tracking each individual producer, household, or government agency in the economy, E-DRAM – like all CGE models - combines similar agents into single sectors. Constructing a cogent sectoring scheme, the first step of model construction, is discussed immediately below; this discussion is followed by a description of the key agents in the economy—producers and consumers.[6]

3.1. E-DRAM components

3.1.1. Aggregation; firms and households

In E-DRAM, the California economy is divided into 186 distinct sectors: 120 industrial sectors, 2 factor sectors (labor and capital), 9 consumer good sectors, 8 household sectors, 1 investment sector, 45 government sectors, and 1 sector representing the rest of the world.

Producers, also known as firms, are aggregated into industrial sectors, and each sector is modeled as a competitive firm. For instance, the output of all of California's agricultural firms is modeled as coming from a single entity—the agriculture sector. Each sector takes the price that it receives for its output and the prices that it pays for its inputs (capital and labor, called "factors of production," and other inputs, called "intermediate goods") as fixed. This is the competitive model: Producers assume that their decisions have no effect on prices. Each producer is assumed to choose inputs and output to maximize profits. Inputs are labor, capital, and intermediate goods (outputs of other firms). Thus, the producer's supply of output is a function of price and the producer's demand for inputs is a function of price.

For industrial sectoring purposes, all California firms making similar products are lumped together. The agriculture sector, for example, contains all California firms producing

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[6] This material on E-DRAM has been adapted from several sources of existing documentation, an overview guide to which is contained in the Appendix.

agricultural products, and the output value of that sector is the value of all crops produced by California growers. A sector's labor demand is the sum of labor used by all firms in the sector. These aggregates generally represent the major industrial and commercial sectors of the California economy, though a few are tailored to capture sectors of particular regulatory interest. For instance, production of internal-combustion engines and consumer chemicals are each delineated as distinct sectors at the request of ARB.

Data for the industrial sectors originate from the U.S. Department of Commerce's Bureau of Economic Analysis and are based on the Census of Business—a detailed survey of U.S. companies conducted every five years. The survey contains information about intermediate purchases, factor (labor, capital, land, and entrepreneurship) payments, and taxes. Although quite extensive, the survey only allows inference about groups of firms at the national level. The conversion of national data to updated California data is accomplished using a combination of state-level employment data and estimates from California Department of Finance's econometric modeling.

Households make two types of decisions: They decide to buy goods and services, and they decide to sell labor and capital services. They are assumed to make these decisions in the way that maximizes their well-being (called "utility" in the economics literature). Like firms, they take the prices of the goods that they buy and the wage of the labor that they sell as fixed. In addition to their labor income, households receive dividends and interest from their stocks and bonds and other ownership interests in capital. Households' supply of labor, as a function of the wage rate, is called the "labor-supply function."

Households' demand for goods or services, as a function of prices, is simply called the "demand function." The latter represents the distribution of household spending across the 120 industrial sectors via the nine consumer goods sectors, and is based on analysis of U.S. Bureau of Labor Statistics' Consumer Expenditure Survey data.

Like firms, households are also aggregated in E-DRAM. California households are divided into eight income categories, each corresponding to a California Personal-income Tax marginal tax rate (0, 1, 2, 4, 6, 8, 9.3, and a high-income 9.3 percent). Thus, for example, the income from all households in the 1 percent bracket is added together and becomes the income for the "1 percent" household sector. Similarly, all expenditure on agricultural goods by the 1 percent households is added and becomes the expenditure of the 1 percent household sector on agricultural goods. Total household expenditure on agricultural goods is the sum of expenditures by all eight household sectors. Household income data come from the California Franchise Tax Board Personal-income Tax "sanitized" sample. Data on consumption by income class are derived from national survey data.

3.1.2. Equilibrium

Households and firms interact through two types of markets: factor markets and goods-and-services markets. Firms sell goods and services to households on the goods-and-services markets. Households sell labor and capital services to firms on the factor markets. There is a price in each of these markets. There is a price for the output of each of the 120 industrial sectors. There is a price for labor, called the "wage," and a price for capital services, called the "rental rate." Equilibrium in a market means that the quantity supplied (which is a function of price) is equal to the quantity demanded (which is also a function of price) in that market. Equilibrium in the factor markets for labor and capital and in the goods-and-services markets for goods and services defines a simple general equilibrium system. That is, there are 122 prices (the wage, the rental rate, and one for each of the 120 goods made by the 120 sectors) and these 122 prices have the property that they equate quantities supplied and demanded in all 122 markets. They are market-clearing prices.

3.1.3. Intermediate goods

There are not only final goods-and-services markets but also intermediate-goods markets in which firms sell to firms. A typical example of this would be chemicals sold to agricultural firms. The final output of the chemical industry (perhaps fertilizer) is said to be an intermediate good in the agricultural industry. Here, part of the supply of a firm (chemical industry in the example) is not sold to households but rather to another firm in exchange for revenue. From the other firm's point of view, it buys an input to production from a firm rather than from a household. The expense of buying the input is a cost of production.

3.1.4. Rest of the world

California is an open economy, which means that it trades goods, services, labor, and capital readily with neighboring states and countries. In E-DRAM, all agents outside California are modeled in one group called "Rest of World." No distinction is made between the rest of the United States and foreign countries. California interacts with two types of agents: foreign consumers and foreign producers. Foreign producers sell goods on the (final) goods-and-services markets and on the intermediate markets, i.e., they sell goods to both households and firms. The model takes these goods as being imperfect substitutes for the goods made in California. Agricultural products from outside of California (e.g., feed grains, bananas) are taken as being close to, but not identical to, California-grown products (e.g., avocados, fresh chicken). Foreign households buy California goods and services on the goods-and-services markets, and they and foreign firms both can supply capital and labor to the California economy.

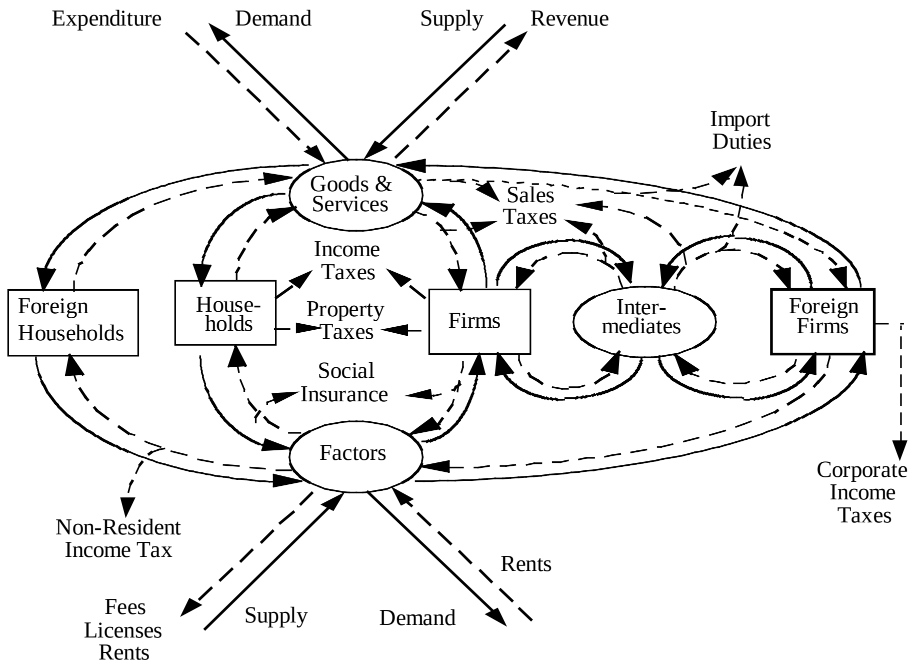
3.1.5. Government

Finally, government is considered. Government collects taxes and supplies goods and services for which it may or may not receive revenue, and also supplies factors of production, such as roads and education. Government also makes transfers to households.

3.1.6. A graphical representation

These relationships are shown in more detail in Figure 1, called a "circular-flow diagram." The outer flows, shown as solid lines, are those of "real" items: Goods, services, labor, and capital. The inner flows, shown as broken lines, are monetary flows. Thus, firms supply goods and services to the goods-and-services market in return for revenues that they receive from the goods-and-services markets. Firms demand capital and labor from the factor markets and in return pay wages and rents to the factor markets. The middle section of the diagram shows ways in which government raises revenue through taxation.

Figure 1. The Circular-Flow Diagram



Source: Berck, Golan, and Smith, 1996.

3.1.7. Data organization: The Social Accounting Matrix

The first step in constructing a CGE model is to organize the data. The traditional approach to data organization for a CGE model is to construct a Social Accounting Matrix (SAM). A SAM is a square matrix consisting of a row and column for each sector of the economy. Each entry in the matrix identifies an exchange of goods and services purchased by one sector from another sector (or itself). The entries along a row in the SAM show each payment received by that particular row sector from each column sector. Summing across the row gives total payments made to that row sector by all column sectors. The entries down a column in the SAM show the expenditures made by that particular column sector to all row sectors. Summing down a column gives total expenditures by that column sector to all row sectors. For accounting purposes, a SAM must "balance," i.e., each row sum and corresponding column sum must be equal. This balancing ensures that no money "leaks" out of the economy, i.e., that all money received by firms (row sum) is spent by them (column sum).

3.2. Modeling future years

Once the major agents in the economy have been identified and the relationship between these agents has been specified, the model is built. In E-DRAM, the algebraic representation of the relationships between the agents in the California economy is achieved with the General Algebraic Modeling System (GAMS) modeling and optimization software. The model currently has 1,100+ equations, exclusive of definitions and of the code to read in and organize the data.

In this analysis, we build upon the method with which EDRAM is applied to deterministic analysis of AB32. In this standard procedure, the California economy in 2020 for base (non-policy) scenarios is represented by scaling the 2003 transactions table (SAM) to 2020 while making adjustments for sectors that are not expected to grow at the same rate as the economy as a whole. This scaling is executed as follows. First, all transactions are multiplied by the ratio of forecasted to current state personal income. Second, specific sectors are adjusted relative to that new base: The energy sectors (OILGAS, OILREF, INDGAS, DISTEL, DSTGAS, WHLGAS, and RETGAS) are adjusted so that their outputs (California produced and refined products) grow at a rate slower than the economy as a whole. This reflects both depletion of California oil fields and the very slow growth relative to SPI of the refining industry. Third, the state population forecasts are included. Characteristically, population growth is also slower than that of the SPI; this difference in growth results in higher productivity within the model. Finally, the new SAM – i.e., for 2020 - is balanced so that each sector has the same income and expenditure.

3.3. Modeling policies: Technological change in E-DRAM

Energy-environment CGE models are designed, and primarily used, for analyzing price-based policies, such as pollutant taxes, emissions cap and trade systems, or low-carbon or energy-efficient technology subsidies. A novel aspect of E-DRAM is that, while it is used in this way, it has also been adapted to incorporate the type of technology-based policies that have been the dominant approach to energy and environmental regulation in California, such as vehicle emissions standards and energy-efficiency performance standards and programs. This is the approach taken to represent in E-DRAM the technology regulation measures that form the core of AB32. The technique used is to model the effects of these measures as a form a technological change, as we now summarize.

As described in Berck and Hess (2000), E-DRAM's underlying structure allows for changes in production technologies as they are abstractly represented in the model. Each industrial sector in E-DRAM is characterized by a production function that relates output to factor (capital and labor) and intermediate inputs. Technological change is modeled by altering the relationships of input mix per unit of output as follows. Industry *J*'s demand for intermediates from industry *I* per unit of output is governed by production parameters AD(*I*,*J*), which are input-output coefficients calculated from primary data contained in the SAM. These coefficients can be altered via technology multiplier parameters REG1(*I*,*J*).

Changing REG1(*I*, *industry J label*) from its default setting of unity to 0.9, for example, simulates a technological change enabling one unit of industrial good *J* to be produced using only 90 percent of the intermediate inputs (from all 120 industries) previously required. Specifying AD(*industry I label*, *industry J label*) = 0.9, in contrast, simulates a technological change enabling one unit of good *J* to be produced using 90 percent of the intermediate inputs previously required from industry *I* (with inputs from the 119 other industries unchanged).

There is a similar utility (REG16) for changing the input requirements for the goods consumed by consumers. For instance, one can reduce the amount of fuel necessary to accomplish a unit of travel, or electricity or natural gas.

3.4. AB32 policy inputs and core ARB Scoping Plan simulations

The method used to construct estimates of GHG-abatement measure costs and benefits for input to E-DRAM is described in Climate Action Team (2007). It is based on an engineering-economic life-cycle cost calculation in which initial incremental capital costs for more energy-efficiency or carbon-reducing equipment are weighed against discounted reduced future operating costs (from lower fuel expenditures). (Operating, maintenance, and other costs are also included as appropriate.) These calculations are extended to incorporate carbon reduction amounts and benefits as well. For input into E-DRAM, these costs and benefits are converted to an annualized form.

These cost and benefit numbers are entered into E-DRAM using the technological change mechanism described above. To take the example of residential energy efficiency programs: Households' utility functions include as an input a fuel aggregate that combines different forms of energy; the programs allow for households' to maintain their given utility levels but with lower expenditures on fuel, which is the model's form of representing energy savings. By virtue of the estimates supplied to the model, the lower expenditures exceed the increased total capital costs necessary to purchase the more energy-efficient technology.

To take another example, vehicle GHG regulations are modeled in a completely analogous way: Lower-emission vehicles cost more initially but less on a life-cycle cost basis because of increased fuel economy. In E-DRAM, as noted above this takes the form of a technological improvement allowing vehicle miles traveled to be obtained at lower cost.

As described in the Scoping Plan, California's Business-as-Usual emissions in 2020 are projected to be 596 million metric tons of CO2 equivalent (MMTCO2E), and the emissions target is 427 MMTCO2E, the estimate of 1990 emissions. The Plan recommends a portfolio of policies and measures to achieve 2020 emissions of 422 MMTCO2E, a slight overcompliance. This portfolio comprises the direct regulatory measures noted above, and an emissions cap-and-trading system. Both the Business-as-Usual projection and the AB32 portfolio were simulated by ARB with E-DRAM to estimate macroeconomic effects, including changes in Gross State Product and Labor Demand as well as aggregate CO2 emissions; the methods and results are documented in CARB (2008a, 2008b, 2008c), and we provide more detail in subsequent sections of this report.

4. Stochastic solutions of E-DRAM

This section describes the technical aspects of implementing a stochastic approach to analyzying AB32 using E-DRAM. We begin by discussing the creation of probability distributions, then summarize the modifications to the model's GAMS code that were made to use these distributions in a Monte Carlo framework to solve the model. We then describe modifications to the procedure that were made to correct problems identified in the initial computational experiments.

4.1. Generating forecast errors and values

As summarized in Section 2.4, for this analysis we focus on the oil price, natural gas price, and economic growth as the uncertain variables. There are several possible approaches to generating statistical forecasts of these variables in 2020. One would be a time-series approach such as vector-autoregression based on historical data series of the variables (Hamilton 1994). For these initial experiments, we take a different tack, and exploit errors in previous forecasts, that is, we use these retrospective errors to generate new prospective error distributions for 2020. The rationale for this approach is that the results of previous forecasts are good guides to how well forecasters can predict going forward. In practice, we found that the publicly-available California-specific forecasts of energy prices and economic growth we were able to identify were conducted with time horizons of no more than three years, whereas for the present analysis we need ten-year forecasts.

Thus, we are initially using the U. S. Energy Information Administration's data on its own ten-year national forecasts of oil and natural gas prices and of the growth rate of U. S. GDP (U.S. EIA 2008). This is based on the hypothesis that the long-run trends in these variables at the national and California state levels are sufficiently similar that the national forecast error patterns are a suitable proxy for direct California observations.[7] Specifically, we have extracted the errors from the EIA's ten-year ahead projections of oil price and natural gas price, and annual projections for economic growth. For oil price (OIL) and natural gas price (NG), these ten-year forecast errors are calculated as:

Percent forecast error = 100*(actual price – predicted price)/actual price, or, equivalently:

$$\varepsilon_i = \frac{(x_{i,t|t} - x_{i,t|t-10})}{x_{i,t|t}} * 100 \qquad \text{where i is either OIL or NG.}$$

The EIA forecasts the annual growth rate of U. S. GDP and the average absolute difference between predicted and actual growth rates. In order to have comparable values to the OIL and NG percent errors, we used the report's data to calculate equivalent percent forecast errors for GDP growth.[8] Percent forecast error of US GDP is used as a proxy for percent forecast error of California personal income because reliable personal income forecast data for California was not available to us.

[7] We were unable to identify publicly-available ten-year forecasts of economic growth and fuel prices specifically for California in sufficient number for our analysis. However, investigation of short-term California forecasts supports our hypothesis that national and state long-run trends are similar to an extent that supports our approach.

[8] Predicted annual growth rates were compounded over a ten-year period and compared to the actual compounded annual growth rates; we then calculated percent forecast error using the same formula: 100*(actual ten-year growth rate – predicted ten-year growth rate)/actual ten-year growth rate.

Table 1. Percent Forecast Errors[9]

Forecast period	OIL	NG	GDP
1985-1995	-189.6	-330.3	1.82
1986-1996	-117.3	-213	15.61
1989-1999	-168.6	-193.7	24.08
1990-2000	-66.7	-45.8	--
1991-2001	-81.1	-7.8	30.38
1992-2002	-81.2	-60.5	27.21
1993-2003	-40.4	1.4	28.61
1994-2004	5.3	29.5	27.70
1995-2005	37.8	41.8	28.91
1996-2006	46.6	55.7	26.96
1997-2007	58.1	56.7	22.93

As an example, if the GDP growth rate was forecast to be 22% over the ten-year period, and the actual growth rate was 25%, then the forecast error would be = 100*(25-22)/25 = 12%.

Tables 2 and 3 contain summary statistics and correlations of the percent forecast errors, which were used to find the means and correlations of the variables using STATA statistical software, which also fit a tri-variate normal distribution to the data. The STATA code after inputting the data is

```
sum OIL NG GDP
cor OIL NG
cor OIL GDP
cor NG GDP
clear
matrix m = (54.28182,60.54545,-23.421)
matrix C = ( 1, 0.9163, 0.5994 \ 0.9163, 1, 0.8397 \ 0.5994, 0.8397, 1 )
drawnorm OIL NG GDP, n(100) corr(C) means(m)
```

As shown in Table 2, GDP growth was, on average, under-predicted, whereas OIL and NG were, on average, over-predicted.

[9] Data are missing for 1987-1997 and 1988-1998 because of a change in methodology by the EIA.

Table 2. Forecast error summary statistics (percent)

Variable	Num. Obs.	Mean	Std Dev	Max	Min
OIL	11	-54.3	84.8	58.1	-189.6
NG	11	-60.5	129.0	56.7	-330.3
GDP	10	23.4	8.7	30.34	1.82

Table 3. Forecast error correlations

	OIL	NG	GDP
OIL	1	0.92	0.60
NG	0.92	1	0.84
GDP	0.60	0.84	1

To illustrate the scale of the errors, Table 4 gives examples of how mean forecast errors could affect the forecasted values:

Table 4. Examples of scale of errors

Variable	Value in 1985	Value in 1995	Mean percent error	Value variable would have taken if predicted with mean percent error	Difference in actual and predicted values
OIL	$26.99/bl	$17.14	54.3	$36.58	-$12.87
NG	$1.74/thousand cu. ft.	$2.95	60.5	$4.74	-$1.79

4.2. Stochastic simulations: Basic approach

In this section we describe the changes made to E-DRAM to accept the probabilistic inputs – that is, samples from the distribution just described - and the execution of the simulations.

The usual policy analyses and experiments with EDRAM do not incorporate moving bases, i.e., multiple projections of the “non-policy” 2020 California economy. Thus, the results of our experiments in this analysis differ from those that are usually reported in that they do not have a single 2020 base case. Rather, the size of the economy in 2020 is a part of the experimental design. We return to this point in sub-section 4.3 and in section 5.

We learned during the course of our initial experiments that the E-DRAM did not contain a complete set of the the emissions projected addressed by the Scoping Plan, and that the omitted emissions were added offline by ARB staff to E-DRAM output in reporting results (CARB 2009). To maintain consistency with ARB's procedure, we added these adjustments to our E-DRAM simulations: 136 MMTCO2E for all Business-as-Usual runs, and 103 MMTCO2E for AB32 runs.

4.2.1. Changing prices

The world price of oil and natural gas affect California through exporting and importing. When the world price of an item increases, E-DRAM will predict more exports and fewer imports. The world price also affects the intra-state price level, because a higher world price leads to a higher price for consumers. We have focused on oil and natural gas. When the world price of oil increases, so does the price of refined product and so we have incorporated an appropriate change in the world price of refined product as well. Details on how these prices were changed are given below.

4.2.2. Programming changes

Conceptually, the basic Monte Carlo simulations were executed as follows. In a given experiment, *N* random draws were first made from the joint distribution function for ten-year ahead forecast error percentage of oil price, natural gas price and California personal income growth. Each draw was then used to generate a different 2020 baseline, without AB32 policies or measures, as a function of the uncertain inputs, via *N* E-DRAM simulations (one for each of the *N* draws) with the values of GSP (Gross State Product), LD (Labor Demand), and CRBCP (CO2E emissions) recorded for each run. Once this was accomplished, the series of *N* simulations was repeated but with the AB32 measures and CO2 price included for each run, and the output values again recorded.

The mechanics of running the simulations focused on creating a loop mechanism within the E-DRAM code. A major limitation on program organization imposed by the structure of GAMS is that one cannot include parameter, acronym, set, file, table, model, equation, variable or scalar statements or equation declaration inside a loop statement. We therefore reorganized the E-DRAM code to put all the declarations outside the loop.

To implement the loop, we created a set NDRAWS, which contains integers from 1 to *N*. We also created a data file DRAWS.PRN, which contains the *N* draws, each for one row. Also, the corresponding parameter DRAWS is declared. Thus, the abstract outline of the code is:

```
Declarations
        Loop (NDRAWS, etc.)
                Run E-DRAM
                Save results
                Repeat
        End Loop
```

Our draws are ten-year ahead forecast error percentage, x%: If x is greater (smaller) than 0, it means it is under-forecasted (over-forecasted) by x%. Thus parameter values are re-set in each loop by multiplying them by (1+x%).

A. *California personal income growth* works through the program Futureyear.gms, which is called first in each loop and makes a 2020 SAM, by assigning a new value to the corresponding parameter INC_GROWTH_FAC. The code is:

```
INC_GROWTH_FAC('%FORECAST_YEAR%')=
INC_GROWTH_FAC('%FORECAST_YEAR%')* (1+DRAWS(Ndraws,'incgrowth'))
```

B. *Oil and natural gas prices* in E-DRAM work through world prices (PW0) of the following industries: Petroleum and Natural Gas Extraction (OILGAS), Natural Gas Distribution (DSTGAS), Oil Refineries (OILREF) and (maybe) Electrical Power Generation and Distribution (DISTEL).

a. OILGAS combines oil and gas, so we use the weighted average of oil and natural gas prices as the price for this industry. The weight is calculated from the SAM. It equals the value from OILREF to OILGAS (18.505) divided by the sum of the value from DISTGAS to OILGAS (7.71896) and the value from OILREF to OILGAS (18.505), which is 0.705652388. So, the code to change the world price of OILGAS is:

```
PW0('OILGAS')
= PW0(' OILGAS ')*(1+0.705652388*DRAWS(NDRAWS,'OILPRICE')+
0.294347612*DRAWS(NDRAWS,'NGPRICE')));
```

b. The price for DSTGAS is obtained by multiplying the original price calculated from SAM (PW0 (DSTGAS)) by (1 + x%). The code is:

```
PW0('DSTGAS')=PW0("DSTGAS ')* (1+DRAWS(NDRAWS,' NGPRICE '));
```

c. The price of OILREF is calculated on the basis of the percentage of refinery output that is oil. This is calculated from SAM and equals the value from OILREF to OILGAS (18.505) divided by the total value from OILREF (53.14193), which is 0.348218441. The code is:

```
PW0('OILREF')
= PW0('OILREF')*(1+ DRAWS(NDRAWS,'OILPRICE') * 0.348218441);
```

C. To save time in running the code and to get a clearer output table, we suppressed all the outputs except three of most interest: gross state product (GSP), labor demand (LD) and carbon emissions (CRBCP). Each is listed for *N* draws for both the Business-as-Usual and AB32 Policy cases, and then used to generate output tables and graphics.

4.3. Stochastic simulations: Trial results and modifications

In our first experiments, we examined two approaches to using the forecast information. First, to focus on the effect of forecast uncertainty solely through the variance of the joint distribution, we demeaned the fuel price and GSP forecasts before before being applied to E-DRAM. That is, in the sample of *N*=100 draws, the sample means of each of the three variables were calculated and subtracted before running the Monte Carlo loop. Thus, the errors had mean zero and the outputs from both the Business-as-Usual simulations and the AB32 measure simulations were centered around the values of the Scoping Plan analysis (after applying the offline scaling noted previously), 596 and 421 MMTCO2E in the BAU and Policy simulations, respectively.

Second, to take account of both uncertainty through forecast variance and "bias" through the under- or over-prediction of the variables as described in Section 4.1, we ran a number of simulations without demeaning. In these cases, particularly because of the bias in the GSP forecast, neither the BAU nor the AB32 simulation outputs were centered around their Scoping Plan values.

A random sample of 100 from the joint error distribution is summarized in Table 5.

Table 5. Forecast error sample statistics

Variable	*N*	Mean	Std. Dev.	Min	Max
GDP	100	0.24	0.09	-0.01	0.44
OIL	100	-0.48	0.82	-3.04	1.59
NG	100	-0.54	1.30	-4.01	2.29

The large standard deviation of the price forecast errors results in a fairly high probability that, when used to perturb E-DRAM prices as described above, a negative price will result, causing a model infeasibility. Since the sample means of the price variables are negative, this problem is mitigated by demeaning. We also found that the price variable uncertainty contributed a relatively small proportion of the output variance in GSP, LD, and CRBCP relative to the uncertainty in the personal income input. Thus, in our main analysis, we used demeaned price errors and undemeaned economic growth errors. In the sample we used, dropping draws with either price error less than -1 reduced the sample size from 100 to 77. We also found that, even with demeaned prices, very low prices could result in an increase of imported goods (to California) sufficiently large that domestic supplies were forced negative, again causing a model infeasibility. Dropping two such draws gave us a sample size of 75, which was used for the stochastic E-DRAM analysis; this sample is summarized in Table 6.

Table 6. Summary of forecast error sample used for analysis

Variable	*N*	Mean	Std. Dev.	Min	Max	Correlations		
						GDP	Oil	NG
GDP	75	0.27	0.07	0.13	0.44	1	0.43	0.79
OIL	75	0.34	0.60	-0.65	2.07	0.43	1	0.86
NG	75	0.56	0.89	-0.90	2.83	0.79	0.86	1

5. Uncertainty analysis of AB32

This section is organized as follows. We begin by summarizing key details of the analytical approach used by ARB to assemble the combined portfolio of regulatory measures and emissions trading recommended in the AB32 Scoping Plan. We then present the key results of the E-DRAM Monte Carlo simulations based on the approach described in Section 4; these are the results prior to undertaking the uncertainty analysis previewed in Section 2. This analysis is presented in Section 5.3.

5.1. Overview of ARB AB32 integrated analysis

As noted in sub-section 3.4, the AB32 Scoping Plan recommends a combination of a) energy efficiency, direct emissions reduction, and other regulatory measures, and b) an emissions cap and trade system, to reduce 2020 emissions from the 596 MMTCO2E projected in the Business-as-Usual case to 422 MMTCO2E. The manner in which the regulatory measures and the cap and trade system were combined is described in the final Scoping Plan (CARB 2008b, section II, “Recommended Actions). In summary, the Plan distinguished two types of sectors: “Uncapped,” subject only to regulatory measures, and “Capped,” subject to both regulatory measures and the cap and trade system. For the uncapped sectors, emissions reductions of 27.3 MMTCO2E were proposed; for the capped, reductions of 112.3 MMTCO2E were proposed from specified measures, with an additional 34.4 MMTCO2E of reductions achieved from the cap and trade system.

Analytically, the cap and trade system was modeled in E-DRAM by imposing a carbon price at the level needed to achieve the desired reductions; this is a standard technique in CGE modeling of GHG policy. For the adopted Scoping Plan, this price was $10 per ton (CARB 2008b, Section III- A).

Conceptually, this approach can be thought of as combining “wedges” of reductions achieved by regulatory measures with a “wedge” achieved by the cap and trade system, allowing as needed for additional iterations as additional direct measures become cost effective with the price incentive introduced by emissions trading.

5.2. Stochastic E-DRAM simulations: Initial results

The purpose of the first Monte Carlo simulations, based on forecast error sample described in Section 4.3, was to introduce uncertainty into the Business-as-Usual and AB32 policy scenarios without making any adjustments to the policy portfolio. As described in Section 4.2.2, for each draw from the sample, E-DRAM was run twice – once to generate a Business-as-Usual scenario corresponding to the input values resulting from the draw, and again with the AB32 portfolio imposed, including the $10/ ton carbon price. In particular, at this stage no adjustments were made to either the measure list or deployment, or the carbon price. The previously-noted scaling of emissions to include sources not represented in E-DRAM was maintained in all simulations, both BAU and Policy: 136 MMTCO2E in BAU, and 103 MMTCO2E in Policy.

The results are presented in Table 7 and depicted in Figures 2, 3 and 4. The figures show both histograms and kernel density estimations. Note that, on average – i.e., with respect to mean values – the macroeconomic result of slightly increased GSP and labor demand in the policy case is preserved. We also re-iterate that, because of the higher mean GSP in 2020 resulting from the economic growth forecast bias, neither the mean BAU nor the mean AB32 (policy) carbon emissions correspond to those of the ARB's simulations – they are both higher.

Table 7. Output statistics for E-DRAM stochastic (Monte Carlo) simulations*

Variable	*N*	Mean	Std. Dev.	Min	Max
GSP – BAU	75	3237	153	2892	3589
GSP – AB32	75	3256	158	2905	3634
LD – BAU	75	18.3	0.181	17.8	18.6
LD- AB32	75	18.4	0.125	18.1	18.6
CRBCP – BAU, Un-scaled	75	527	20	482	579
CRBCP – AB32, Un-scaled	75	393	19	358	440
CRBCP – BAU, Scaled	75	663	20	618	715
CRBCP – AB32, Scaled	75	496	19	461	543

*Units: GSP, billions of dollars; LD, millions of jobs; CO2E emissions (CRBCP), millions of metric tons of CO2 equivalent.

Figures 2a and 2b. California Gross State Product (GSP) in 2020-distributions in Business-as-Usual and AB32 policy simulations

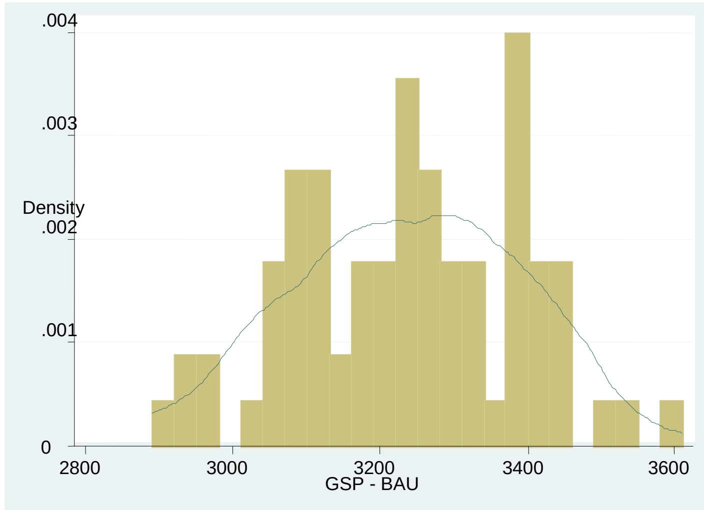


Figure 2a – Year 2020 BAU case California GSP (billions of dollars)

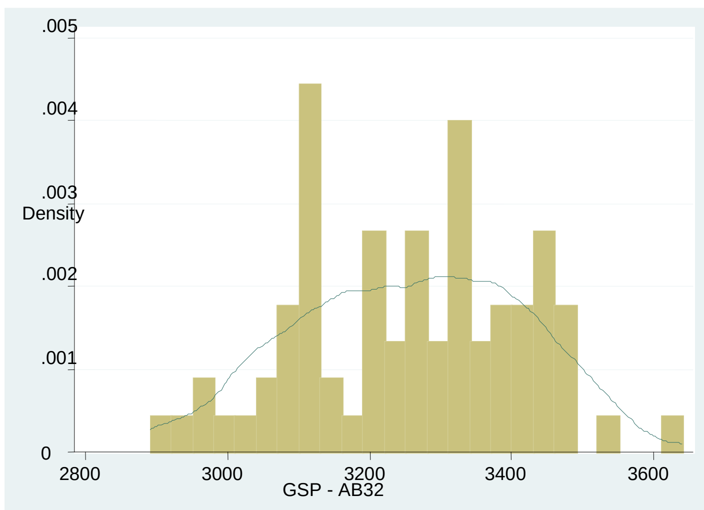


Figure 2b – Year 2020 AB32 case California GSP (billions of dollars)

Figures 3a and 3b. California labor demand (LD) in 2020- distributions in Business-as-Usual and AB32 policy simulations

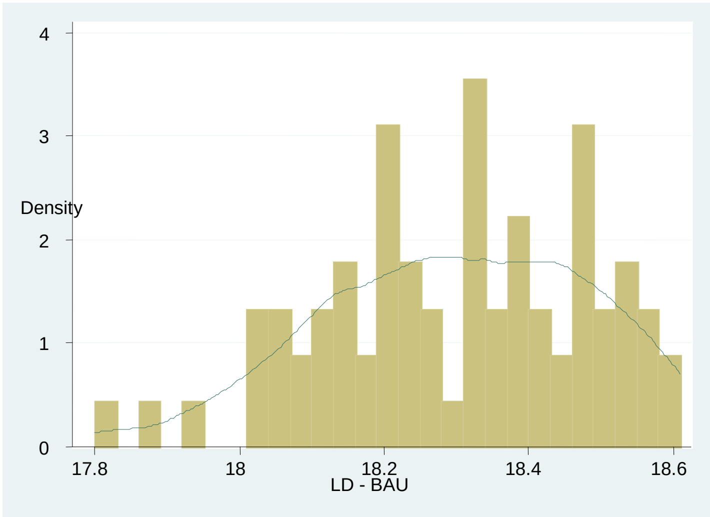


Figure 3a - Year 2020 BAU case California labor demand (millions of jobs)

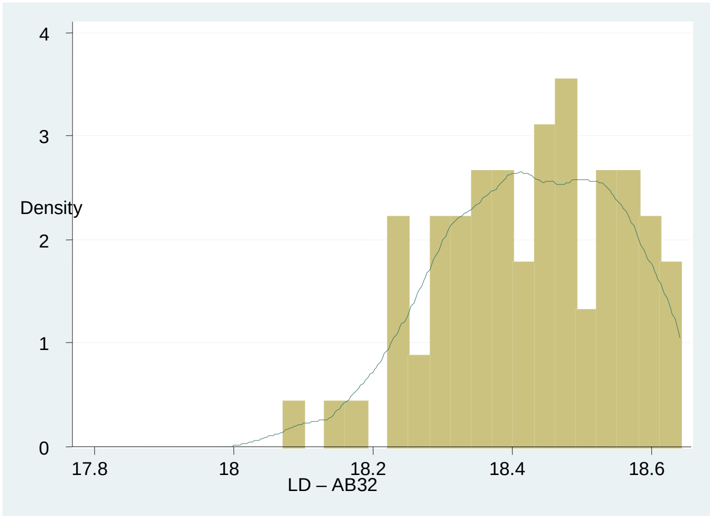


Figure 3b - Year 2020 AB32 case California labor demand (millions of jobs)

Figures 4a, 4b, 4c. California CO2 emissions in 2020 - distributions in Business-as-Usual and AB32 policy simulations

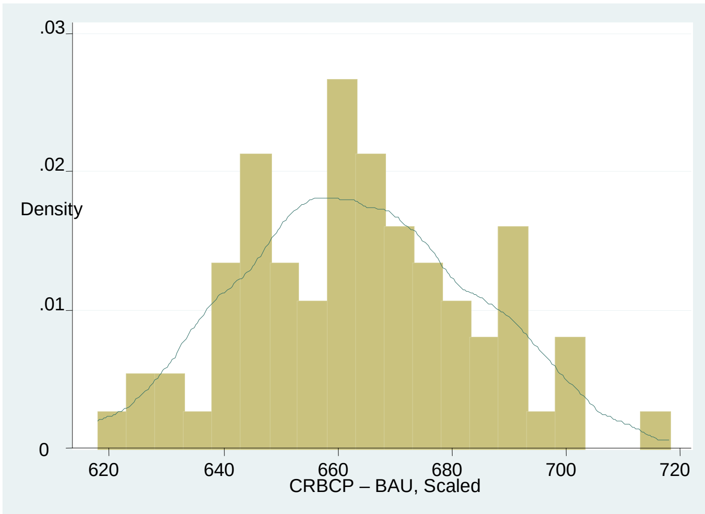


Figure 4a – Year 2020 BAU case California aggregate GHG emissions (MMTCO2E)

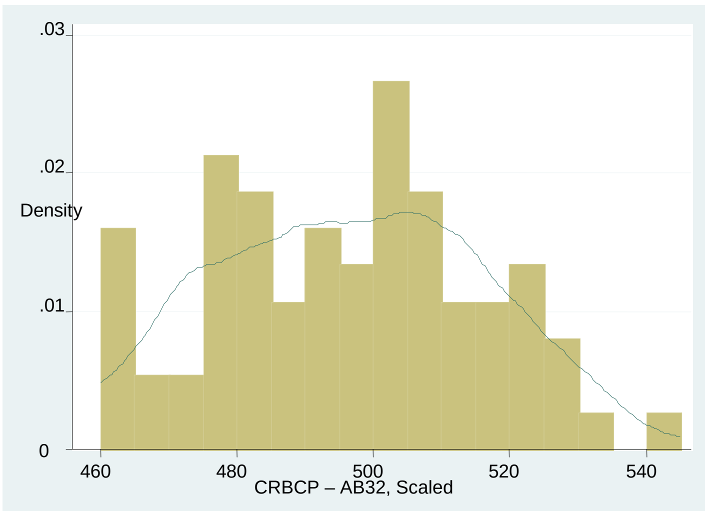


Figure 4b – Year 2020 AB32 case California aggregate GHG emissions (MMTCO2E)

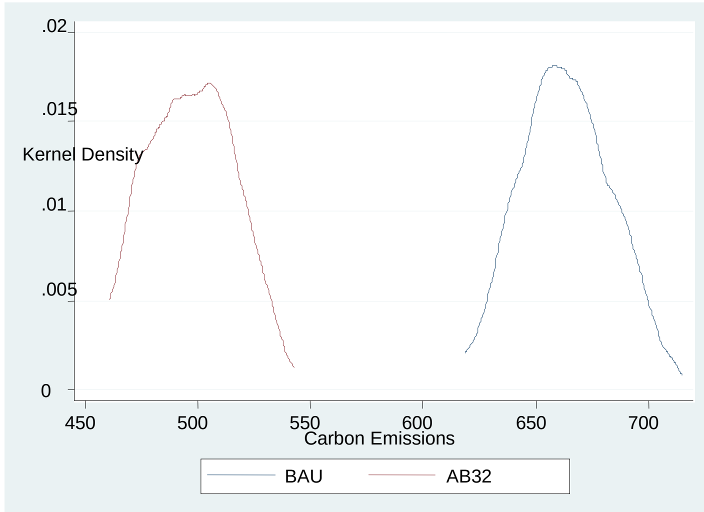


Figure 4c – Kernel densities of year 2020 BAU and AB32 emissions

5.3. Emissions policy under uncertainty

5.3.1. Framing the problem

The discussion in Section 2 provides a framework for interpreting these results and for considering how to apply them. First, these stochastic E-DRAM simulations are a counter-example to the idea that deterministic incremental analysis obviates problems associated with uncertainty in model-based CO2 policy analysis. We have added uncertainty to the E-DRAM projections while maintaining the emissions-reduction "wedges" of the Scoping Plan portfolio, and therefore preserved the incremental structure of the ARB's analysis. However, as we previously argued, with the policy goal – in this case the AB32 emissions target - set as an absolute rather than relative or incremental target, this is insufficient to make the original analysis robust to the uncertainty, because across a wide segment of the projected 2020 CO2 distribution the target is missed. Thus, taking account of the uncertainty requires some form of additional analysis.

Second, both the uncertainty from forecast variance and that from forecast bias contribute to this problem. Had we demeaned the economic growth errors, so that the 2020 CO2 distribution was centered (approximately) around the ARB's projections both in the BAU and in the AB32 scenarios, then the results could be interpreted, roughly speaking, as showing that the expected outcome of the Scoping Plan portfolio would be that the 2020 target was met. However, concluding from this that the uncertainty had been successfully accounted for would be erroneous because the two countervailing *risks* would have been ignored: On the one hand, under-mitigating and missing the 2020 emissions target; on the other, over-mitigating and thus incurring greater costs than necessary to meet the target. Thus, even when the mean outcomes under uncertainty correspond to the original deterministic outcomes, the introduction of uncertainty poses an unavoidable trade-off.

When, as in our E-DRAM simulations, there is a deviation between the deterministic results and the mean of the stochastic results, then there is a skewing of these relative risks. In this case, there is a higher probability of missing the emissions target than of over-reducing at excessive cost, because the bias in the economic growth forecast errors results in mean 2020 BAU emissions higher than ARB's projection.

As we also noted in Section 2, from an abstract theoretical perspective how this information is used to guide policy depends on the policy-maker's loss or utility function. In principle, this function would summarize the policy-maker's preferences regarding the factors we have discussed: How to make the trade-off between emissions reductions and cost containment, and how to weigh the relative risks associated with these two criteria. In other words, this function would serve as an optimization criterion, and with further E-DRAM simulations – changing the list of regulatory measures and/or the carbon price – it would enable the calculation of an optimal policy by minimizing expected loss or equivalently maximizing expected utility.

However, this framework provides little or no practical guidance here, if only because AB32 reflects the views and judgements of many individuals and entities, combined in a lengthy and complex process. There is no apparent way of determining an appropriate loss function to apply.

We instead address the problem pragmatically in terms of the concepts of inverse analysis and robustness discussed in Section 2. In terms of the "dual" approaches we described there, we focus on adjusting the CO2 policy to meet the 2020 target with varying likelihoods (rather than adjusting the CO2 target to meet a given cost level). Specifically, we experiment with different carbon prices in E-DRAM simulations, increasing the carbon price "wedge" while retaining the core AB32 package of regulatory measures. While the changed carbon price results in some increased uptake of regulatory measures as a result of increased cost-effectiveness, the primary effect is due to the price; as we noted previously, this is a proxy for tightening the cap in the proposed emissions cap and trade system.

This is an inverse approach in the sense that we are holding the CO2 emissions target fixed and then working "backward" to determine the policy that will achieve the target, in this case given different resolutions of the forecast uncertainty. As we further discuss below, the robustness concept can be viewed in this analysis as a criterion for gauging, prior to this resolution, the likelihood, but not certainty, of meeting the target.

5.3.2. E-DRAM computations

Our starting point is the distribution of projected 2020 CO2 emissions as summarized in Table 7 and shown in Figure 4. The mean BAU emissions are 663 MMTCO2E, and the mean emissions under the Scoping Plan portfolio of regulatory measures and $10 carbon price are 496. Table 8 shows emissions corresponding to three quantiles of the distribution of emissions under AB32, and the increment by which emissions at each quantile exceed the target of 421. That is, for the three levels of carbon emissions - 482, 499 and 512 - the likelihoods that carbon emissions are larger in this distribution are 75%, 50% and 25% respectively, and the respective amounts by which they exceed the target are 61, 78, and 99 MMTCO2E.

Table 8. 2020 carbon emissions (in MMTCO2E) under Scoping Plan policies: Three quantiles of distribution

Quantile	Emissions	Emissions excess over AB32 target
75%	512	99
50%	499	78
25%	482	61

Source: E-DRAM output.

To carry out the analysis, it would be possible to calculate, for each of the E-DRAM simulations corresponding to the 75 draws in the error distribution, the price needed to achieve the emissions target of 421. However, this would have entailed three-to-four hundred separate runs of the model, the vast majority of which would have provided no real additional information. For this reason and for the sake of tractability, we instead focused on the three quantiles shown in the table.

We proceeded as follows. Each of the three quantile emissions levels shown in Table 8 was the output of an E-DRAM simulation based on a specific draw from the joint input distribution of GDP, oil price, and natural gas price forecast errors. For each of these draws in turn, we re-ran E-DRAM with successively higher carbon prices – in increments of \$10/ton – until the aggregate emissions were below the target of 421. In all these simulations we maintained the AB32 package of regulatory measures. The results of these computations are shown in Table 9. The first column shows the particular error draw that was associated with each emissions quantile. The second column shows the carbon price increments, and the third the incremental reduction in total emissions associated with each; recall that the \$10 price was part of the AB32 portfolio, and so the first entry for "total emissions" in each group corresponds to the values in Table 8.

Table 9. Error draws and associated carbon prices and emissions decreases for three quantiles

Quantile, error draw – (GDP, OIL,NG)	Carbon price	Emissions decrease	Resulting total emissions under
First, (0.23,1.06,1.78)	$10	0	482
	$20	13	469
	$30	30	452
	$40	43	439
	$50	57	425
	$60	69	413
Second, (0.28,-0.36,0.06)	$10	0	499
	$20	29	470
	$30	59	440
	$40	82	417
Third, (0.31,-0.11,0.33)	$10	0	512
	$20	25	487
	$30	51	461
	$40	73	439
	$50	94	418

Source: E-DRAM simulations

As shown in the table, in each case, as expected, increasing the carbon price results in an emissions decrease relative to the starting level. It might also be expected that the higher the starting level of emissions, the greater the carbon price ultimately required to bring total emissions below the cap. This turns out not to be the case, however. For the first quantile carbon emissions, a price of $60 is required to decrease emissions by 69 MMECO2E that brings total emissions below the target. For the second quantile, these values are $40 and 82 MMTCO2E, respectively, and for the third, $50 and 94 MMTCO2E.

The reason for this seeming anomaly is that, although the successively higher emissions quantiles are associated with correspondingly higher economic growth inputs, the oil and gas price inputs are not similarly ordered. Oil price is lower in the second quantile draw than in the first, between the two in the third; the same pattern holds for the natural gas price. Thus, the results show that the response of the economy to carbon prices, and therefore the magnitude of the price required (in conjunction with the regulatory measures) to meet the 2020 emissions target depend not only on the size of the "gap" between projected and target emissions, but also on the particular combination of economic growth and price forecasts.

To explore this further, we repeated the analysis for additional draws from the input error sample, clustered around the three emissions quantiles. First, for each quantile we picked out four additional E-DRAM simulations that yielded output emissions at or near the same level, but that were generated with different draws from the input error sample. These twelve selections are summarized in Table 10.

Table 10. Stochastic inputs and resulting emissions under AB32 policies

		Draws from input forecast error joint distribution			Emissions (MMTCO2E)
		GDP	OIL	NG	
	Draw number				
Around 1st quantile	1	0.20	0.68	0.75	480
	2	0.21	-0.20	-0.38	477
	3	0.20	0.00	-0.16	480
	4	0.21	0.64	0.48	482
Around 2nd quantile	5	0.25	0.15	-0.06	496
	6	0.30	1.32	1.64	499
	7	0.29	-0.53	0.04	497
	8	0.28	0.92	1.25	500
Around 3rd quantile	9	0.31	0.68	1.22	509
	10	0.31	0.56	1.16	510
	11	0.32	0.79	1.08	513
	12	0.31	-0.19	0.74	512

Source: E-DRAM simulations

As in the first set of simulations, for each of these twelve draws we ran E-DRAM with successively increasing carbon prices – in increments of $10/ton – until the model projected 2020 emissions at or below the AB32 target of 421. As before, the “starting emissions” are those projected by E-DRAM with the Scoping Plan AB32 package of measures and $10/ton price. The results are illustrated in Table 11. In keeping with the previous finding, this carbon price threshold can vary substantially between simulations with almost identical initial emissions levels. For example, starting emissions from draws 6 and 7 differ by only 2 MMTCO2E but the carbon price needed to meet the target is $80 with draw 6 but $40 with draw 7. This is again due to the fact that in these simulations, while economic growth assumptions are very similar, the fuel price assumptions are quite different (as show in Table 10), and so the model responds very differently to the carbon price.

Table 11. Carbon prices (in increments of $10/ton) needed to meet AB32 emissions target as a function of input forecast error draw

	Draw number	Starting emissions (MMTCO2E)	Minimum carbon price to meet AB32 target ($/ton)
1st quantile	1	480	$50
	2	477	40
	3	480	40
	4	482	50
2nd quantile	5	496	50
	6	499	80
	7	497	40
	8	500	70
3rd quantile	9	509	70
	10	510	70
	11	513	70
	12	512	60

Source: E-DRAM simulations

5.3.3. Interpreting the results and developing robust policies

We previously defined "robustness" as a criterion for assessing policies under uncertainty and an alternative to optimization criteria – when, for example, policy-makers' loss or utility functions are not explicitly available. To re-state with this terminology a point made previously, the Scoping Plan portfolio is not robust with respect to the uncertainty we have examined here because both mean emissions, and most of the projected emissions distribution, are above the target level.

We can use our results to illustrate how robust policies could be characterized, conditional on judgements regarding the level of certainty that is desired as well as the trade-off between emissions reduction and cost containment. That is, although we do not have a mathematical loss/utility function, subjective judgements of some form are unavoidable. (While the following discussion is of course based on our use of E-DRAM and the particular results we have generated, including our treatment of forecast errors, the underlying themes are more generally valid.)

Suppose, for example, that maximizing the likelihood of meeting the 421 target regardless of cost is the sole goal. Although our E-DRAM simulations do not encompass this case, in principle this could be achieved by adding additional regulatory measures and raising the carbon price sufficiently that the target would be reached from the maximum projected BAU emissions of 715 MMTCO2E, i.e., a reduction of 294 MMTCO2E, nearly seventy percent greater than the reductions proposed in the Scoping Plan. (We do not know if it would be possible to simulate this with the existing ARB measures and underlying price responsiveness of E-DRAM. Note that the greatest emissions reduction in our experiments was 173 MMTCO2E from BAU, which occurred with an $80/ton carbon price in the simulation based on error draw number 6.)

This policy would be highly robust to the uncertainty we have modeled with respect to meeting the AB32 target. However, even with different numbers, including error projections, and projections from models other than E-DRAM, we think it likely that in practice this type of scenario would reveal that many if not most stakeholders would indeed be willing to accept some risk of missing the target in order to keep abatement costs within bounds, if some significant level of emissions reduction is achieved nevertheless. Among other considerations, this type of "meeting the target at all costs" approach involves very significant costs to hedge against outcomes with very small probabilities.

Trading off some risk of excess (over-target) 2020 emissions for a lower cost of abatement could take the form of setting desired likelihoods or confidence levels and then computing the different policy portfolios require to meet them. These levels would take the form of, for example, a 75%, or 90%, probability of meeting the 421 target. In effect, these thresholds would define robustness criteria, and the derived portfolios would associate a cost for meeting a given threshold. Were there a one-to-one relationship between policy portfolios and emissions, this procedure would identify a unique portfolio corresponding to each confidence level, and by computing the costs of each portfolio, we could fully describe the risk trade-off faced by our hypothetical decision-maker, and what preferences might lead to different choices about how to make this trade-off.

However, the results of the previous sub-section complicate this analysis, for they imply that the policy portfolio for each confidence level will not be uniquely determined. To understand why, suppose that our uncertainty analysis had included a single uncertain forecasted input, say economic growth. We know that, all else being equal, E-DRAM's projected CO2 emissions are increasing in the size of the state economy. Thus, each quantile of the 2020 emissions distribution would correspond to a single value in the growth forecast distribution, and therefore we would be able to identify a single carbon price that, for the given realization of the forecast error, would ensure that the 421 target would be met. Moreover, the required price would be monotonically increasing in both forecast economic growth and associated projected emissions; that is, the greater the "gap" between projected and target emissions, the greater the price. Therefore, for a given confidence level as described in the previous paragraph, we could calculate a unique policy and thus unique associated cost, and completely characterize the emissions/cost risk trade-off as a function of the confidence or robustness threshold.

By contrast, as we discussed in the preceding sub-section, with our multi-variate error forecast given emissions reductions are associated with multiple draws and multiple policies, as shown in Table 11. (Although we only dealt with three quantiles, there is no reason to infer that this pattern would be different in other regions of the emissions distribution.) This implies that defining the risk trade-off as we have discussed entails solving, for any desired level of confidence, a form of inverse problem with multiple solutions.

The computations we have discussed in this paper do not provide sufficient information to carry out this analysis. To do so would require a significantly increased number of E-DRAM simulations in order to map out the high-dimensional space of 2020 emissions distributions as a function of the carbon price. With this information, we could investigate solutions to the set of inverse problems just described. It might be the case, for example, that imposing an additional criterion – such minimum cost among portfolios associated with a given confidence or robustness threshold – would reduce the problem dimension sufficiently to characterize the fundamental trade-off in one or two dimensions. (This type of approach is the common one for inverse problems in many disciplines.)

To close this section, we wish to interpret our findings from a slightly different perspective. We have previously argued against the view that incremental deterministic GHG policy analysis is a valid means of dealing with uncertainty when dealing with absolute, rather than incremental, policy goals. This was first illustrated by the initial stochastic E-DRAM simulations, which showed that simply including forecast uncertainty in the BAU and AB32 policy projections implied a high probability that the AB32 2020 emissions target would not be met.

This point is further demonstrated by the results presented in the previous sub-section. In fact, these results reveal a basic limitation of the “wedge” form of GHG policy analysis. One way of dealing with mean projected BAU emissions that are higher than previously projected might be to simply move the deterministic baseline higher and add additional “wedges” in the form of greater deployment of regulatory measures, a higher carbon price, or both. However, our E-DRAM simulations show that, at least for carbon prices, the presence of uncertainty breaks down the fundamental underpinning of the wedge logic, namely the relationship between a given policy action and the resulting emissions reduction. This is illustrated in Table 12, which shows the range of CO2E reductions associated with a given carbon price across the simulations associated with the twelve forecast error draws in Table 11. There is considerable uncertainty in the emissions reduction from *each* price level. In this sense, the concept of a carbon price “wedge” is not well-defined.

Table 12. Emissions reductions from carbon prices – results from E-DRAM simulations based on twelve draws from forecast error distribution

Carbon price	Associated emissions reductions (MMTCO2E)		
	Min	Max	Range
$20	13	29	15
30	28	61	32
40	41	86	45
50	54	111	57
60	66	134	67
70	76	153	77
80	87	173	86

Source: E-DRAM simulations

6. Conclusion

Developing and implementing large-scale GHG emissions abatement policies is arguably the most complex energy and environmental policy challenge the world has faced to-date. The significant progress that has been made thus far, in California, other U. S. states, the U. S. federal level, and around the world, owes much to an analytical knowledge base, developed over several decades, that includes economic modeling and analysis methods.

Going forward, further policy and methodological progress will need to be made in tandem. A host of issues associated with the multiple uncertainties involved in GHG policy have come to attract increasing attention from, as well generate controversy among, a variety of stakeholders. Practical methods for addressing these uncertainties should thus be a focus of research attention.

In this paper we have described the elements of one approach and some initial results of its implementation. This approach allows for leveraging existing model and data resources, rather than requiring they be simplified or suppressed, while also, in our view, yielding results that can be accessible in policy as well as research circles. We believe that further elaboration of this approach can provide policy makers with valuable insights in dealing with the challenges of climate policy.

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Appendix – Documentation of E-DRAM

Fuller description of the common features shared by E-DRAM and DRAM is contained in Berck and Hess (2000). The original DRAM documentation (Berck et al. 1996) contains very detailed information on the structure and construction of the model; its contents are as follows:

- Chapter II: Identification of major agents in the economy, description of aggregation scheme, discussion of data sources;
- Chapter III: Economic behavior of households with respect to consumption and savings decisions;
- Chapter IV: Production decisions of firms;
- Chapter V: International and interregional trade;
- Chapter VI: Investment theory;
- Chapter VII: Regional labor-supply response to taxation and economic growth;
- Chapter VIII: Migration and economic growth.

After establishing the sectoring scheme, data sources, and behavioral equations for the model, all that remains before the actual model can be built is a description of the model-closure rules. Closure rules concern the mathematics of insuring that a solution exists to the 1,100+ equations of the model. Model closure is developed in Chapter IX of the DRAM Report.

Chapter X of the DRAM Report describes the mathematical and corresponding GAMS notation for each equation in DRAM. It is a technical description of the complete California DRAM.[10] Chapter XI presents some preliminary sensitivity analyses.

Appendices include the original literature search by Dr. Berck and Mr. Dabalen in the Summer of 1995, explanations of notational methods used, lists of parameter and variable names used in the mathematical and software input files, and printed copies of the input files themselves.

The updating to the 2003 base year is documented at http://are.berkeley.edu/~peter/Research/DRAM03B/OverviewIII_1018.doc.

The most recent updating is documented at http://are.berkeley.edu/~peter/Research/2003_sam_and_edram.htm.

Particularly, see "Construction of SAM" for technical details and spread sheet models. See SAM120 for the basic models. See "Predicting Future Years" for an explanation of how the future SAMs were calibrated to data on employment, income, etc.

[10] See Berck, Hess, and Smith (1997) for revisions to the consumer demand portion of the model. Modification of equations from DRAM to E-DRAM are discussed in Berck and Hess (2000); the changes introduce parameters that facilitate running policy scenarios as some combination of price, intermediate good, and/or investment changes.