ARE 202: Welfare: Tools and Applications
Spring 2018

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Lecture notes 03 – Applications of Revealed Preferences

Revealed preferences: implications and applications

- WARP application 1: Testing rationality
- WARP application 2: Shape of indifference curves
- WARP application 3: GARP and rationalization
- WARP application 4: Recoverability
- WARP application 5: Laspeyres vs. Paasche price indexes
- WARP application 6: Effect of a tax on welfare
- WARP application 7: Welfare gains from trade
- Side note: Aggregation of WARP

Rationalization: revealed preferences

- **Weak Axiom** of Revealed Preferences (WARP): $x(p, w)$ satisfies WARP if the following property holds for any (p, w) and (p', w'):

$$p.x(p', w') \leq w \text{ and } x(p', w') \neq x(p, w) \quad \Rightarrow \quad p'.x(p, w) > w'$$

Weak Axiom has tons of practical implications for applied analysis of consumer choice

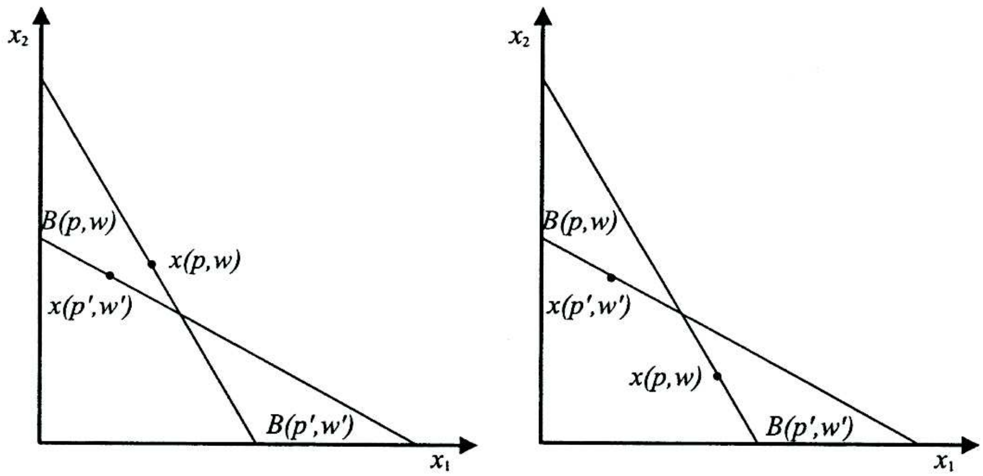


Figure 5.2
(a) Satisfies the weak axiom. (b) Does not satisy the weak axiom.

In other words...

WARP means [finish my sentences]:

- If you choose basket A initially, and now you choose basket B which you could also afford initially, we can deduce that...
- If you choose basket A initially, and now you choose basket B while basket A remains affordable, we can deduce that...
- If you choose basket A initially while basket B is affordable, and now you choose basket B while basket A remains affordable, we can deduce that...

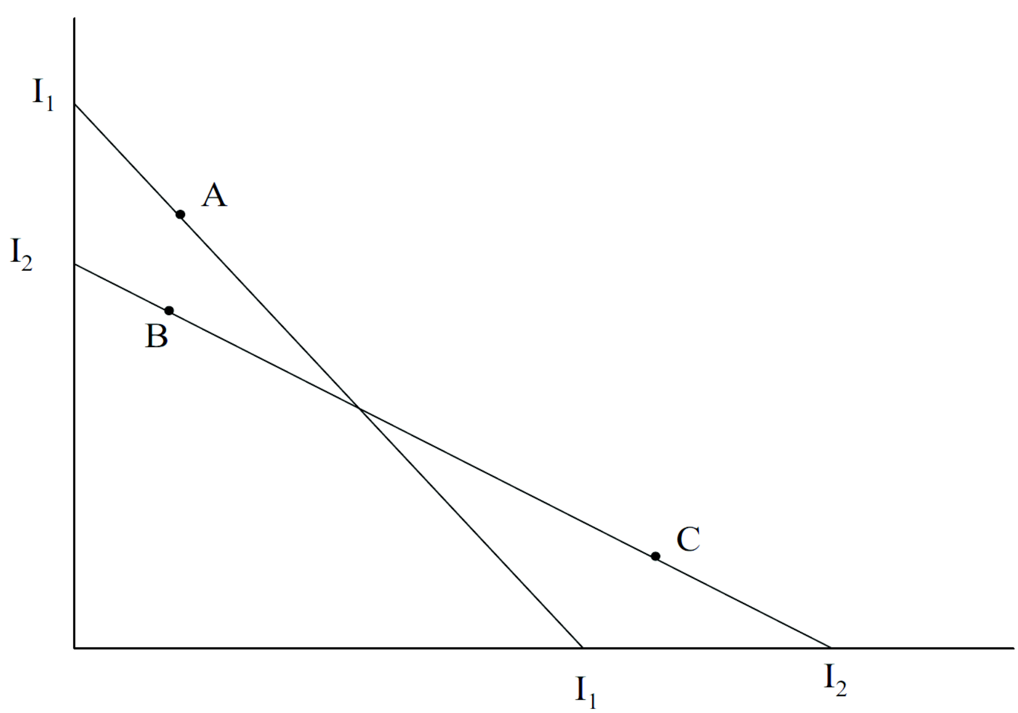
In other words...

WARP means:

- If you choose basket A initially, and now you choose basket B which you could also afford initially, we can deduce that:
  - you can no longer afford basket A.
  - you suffer from a loss of utility.
- If you choose basket A initially, and now you choose basket B while basket A remains affordable, we can deduce that
  - you could not afford basket B initially.
  - you gain in utility.
- If you choose basket A initially while basket B is affordable, and now you choose basket B while basket A remains affordable, you are not rational from an economist's point of view.

- Moving from A to B implies a loss in utility. We say that A is "revealed preferred" to B

- Ambiguous if moving from A to C.



Revealed Preferences: Seven Applications

1. Testing rationality
2. Shape of indifference curves
3. GARP and Rationalization
4. Recoverability
5. Laspeyres vs. Paasche price indexes
6. Effect of a tax on welfare
7. Welfare gains from trade

Application 1: Testing rationality

Are these demand patterns rational?

Consider these consumer choices:

- At prices (p1,p2)=($2,$2) the choice is (x1,x2) = (10,1).
- At (p1,p2)=($2,$1) the choice is (x1,x2) = (5,5).
- At (p1,p2)=($1,$2) the choice is (x1,x2) = (5,4).

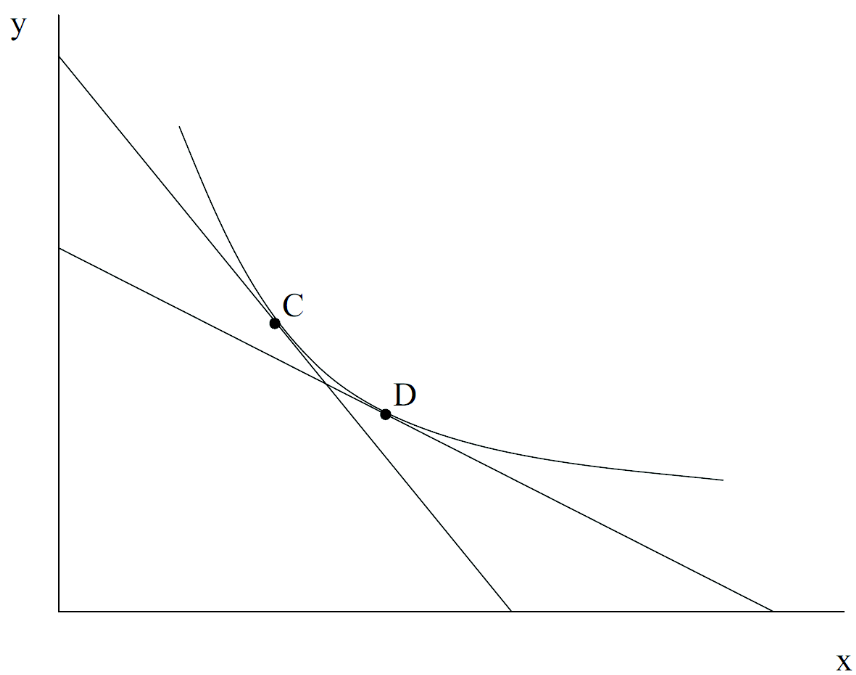
Hint: which bundle is "revealed preferred" to another bundle?

Application 1: Testing rationality

Answer: see blackboard

Application 2: Shape of indifference curves

What can WARP tell us about indifference curves?



Application 2: Shape of indifference curves

In the this figure, when consumer is indifferent between C and D:

- It must be that C is not within the budget set when D is chosen. Reciprocally, D is not in in the budget set when C is chosen.
- Formally, this implies:

$$p_x^C X_C + p_y^C Y_C \leq p_x^C X_D + p_y^C Y_D \quad \text{when C is chosen}$$

$$p_x^D X_D + p_y^D Y_D \leq p_x^D X_C + p_y^D Y_C \quad \text{when D is chosen}$$

Application 2: Shape of indifference curves

- Rearranging, we get:

$$p_x^C (X_C - X_D) + p_y^C (Y_C - Y_D) \leq 0$$

$$p_x^D (X_D - X_C) + p_y^D (Y_D - Y_C) \leq 0$$

- Then, taking the sum, we obtain:

$$(p_x^C - p_x^D).(X_C - X_D) + (p_y^C - p_y^D).(Y_C - Y_D) \leq 0$$

- If we assume that demand is differentiable (holding u constant) and consider only small changes in p_y, this implies:

$$\frac{\partial X}{\partial p_x}|_u \leq 0$$

$\Rightarrow$ WARP implies downward-sloping (compensated) demand.

No need for assumptions on quasi-concavity of U or MRS.

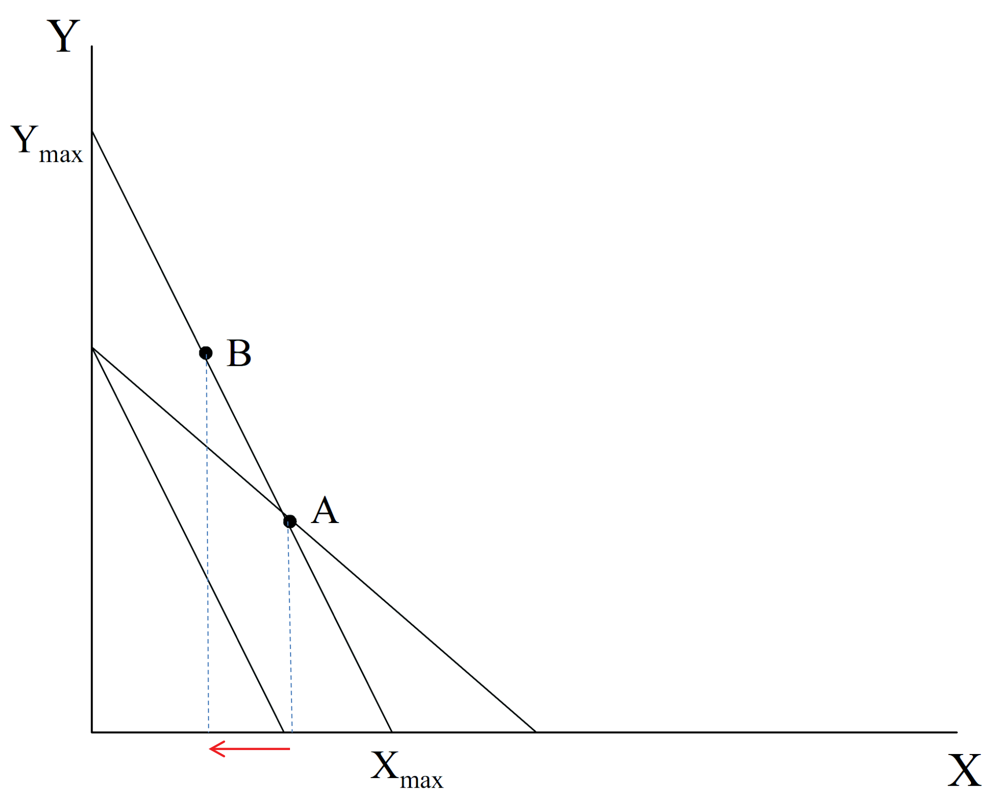
Application 2: Shape of indifference curves

Another proof (see next figure):

- Suppose that we start from A and increase the price of good X
- Suppose also that we "compensate" the consumer such that the new budget line includes A.

⇒ WARP implies that the new demand leads to a decrease in the consumption of X.

Hence, once we neutralize the wealth effect (= compensated demand), the price effect is negative



Application 3: Rationalization

Under which conditions can demand $x(p, w)$ be derived from a preference structure?

Application 3: Rationalization

Under which conditions can demand $x(p, w)$ be derived from a preference structure? We need SARP:

- **Strong Axiom** of Revealed Preferences (SARP): $x(p, w)$ satisfies SARP if the following property holds for any sequence $\{(p_n, w_n)\}$ of prices and budgets:

$$p_n.x(p_{n+1}, w_{n+1}) \leq w_n \text{ and } x(p_n, w_n) \neq x(p_{n+1}, w_{n+1})$$

$$\Rightarrow \quad p_N.x(p_1, w_1) > w_N$$

- Essentially, SARP imposes transitivity in addition to WARP
  - SARP more difficult to check, so most people focus on the Weak Axiom even if it is not sufficient (counter-example involves three goods)

Rationalization vs. Integrability

Slutsky criterium: Hurwicz Uzawa (1972) "Integrability" Theorem

We know that $h_i(p,u) = \frac{\partial e(p,u)}{\partial p_i}$ (Shephard's Lemma) and:

- $S_{ij} = \frac{\partial h_i(p,u)}{\partial p_j} = \frac{\partial h_j(p,u)}{\partial p_i} = S_{ji}$ (symmetric substitution effects)
- The substitution matrix S (with coefficients S_{ij}) is definite negative

$\Rightarrow$ **sufficient** conditions to conclude that a demand function is rational!!
(along with being homogeneous of degree zero and continuously differentiable).

How to obtain S_{ij} from Marshallian demand?

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- The substitution matrix S (with coefficients S_{ij}) is definite negative

$\Rightarrow$ **sufficient** conditions to conclude that a demand function is rational!!
(along with being homogeneous of degree zero and continuously differentiable).

How to obtain S_{ij} from Marshallian demand? Reminder:

$$S_{ij} = \frac{\partial x_i(p,w)}{\partial p_j} + x_j(p,w).\frac{\partial x_i(p,w)}{\partial w}$$

Rationalization vs. Integrability

Slutsky criterium: Hurwicz Uzawa (1972) “Integrability” Theorem

The definite (or semi-definite) negativity of the Slustky matrix is tightly linked to the convexity of indifference curves, itself tightly linked to WARP.

Application 4: Recoverability

Seminal paper: Varian (1982)

Goals:

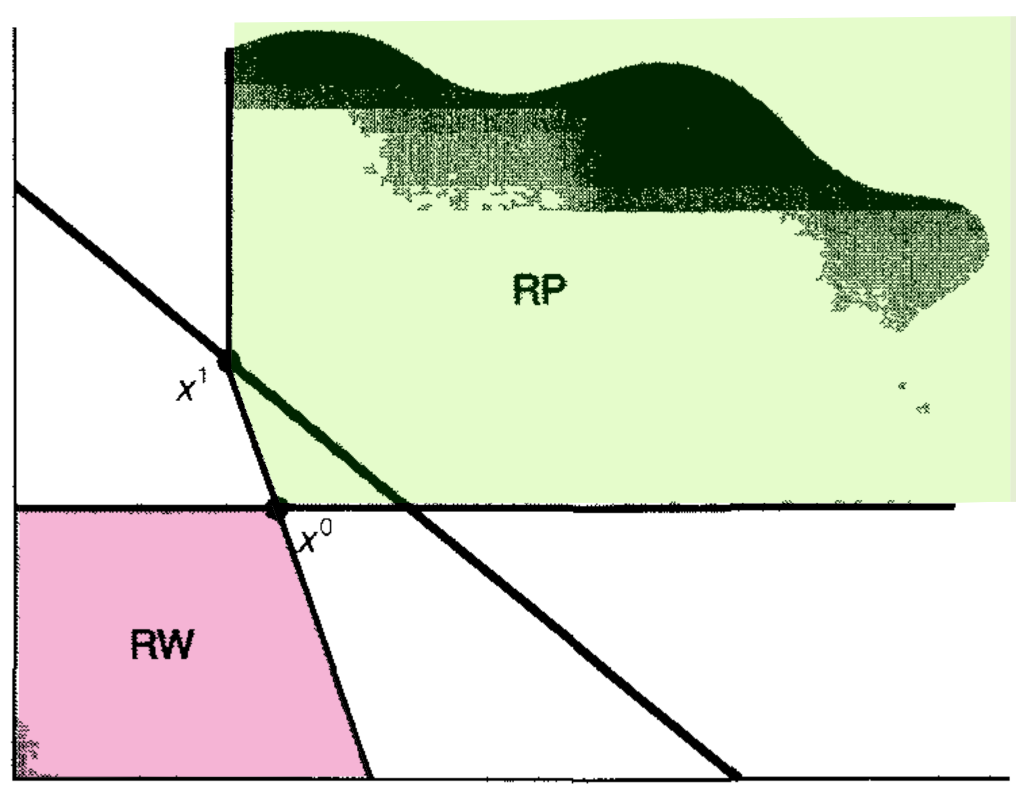
- Use observed choices and WARP to predict indifference curves
- infer preferences among choices that have not yet been observed

Recoverability with one observation

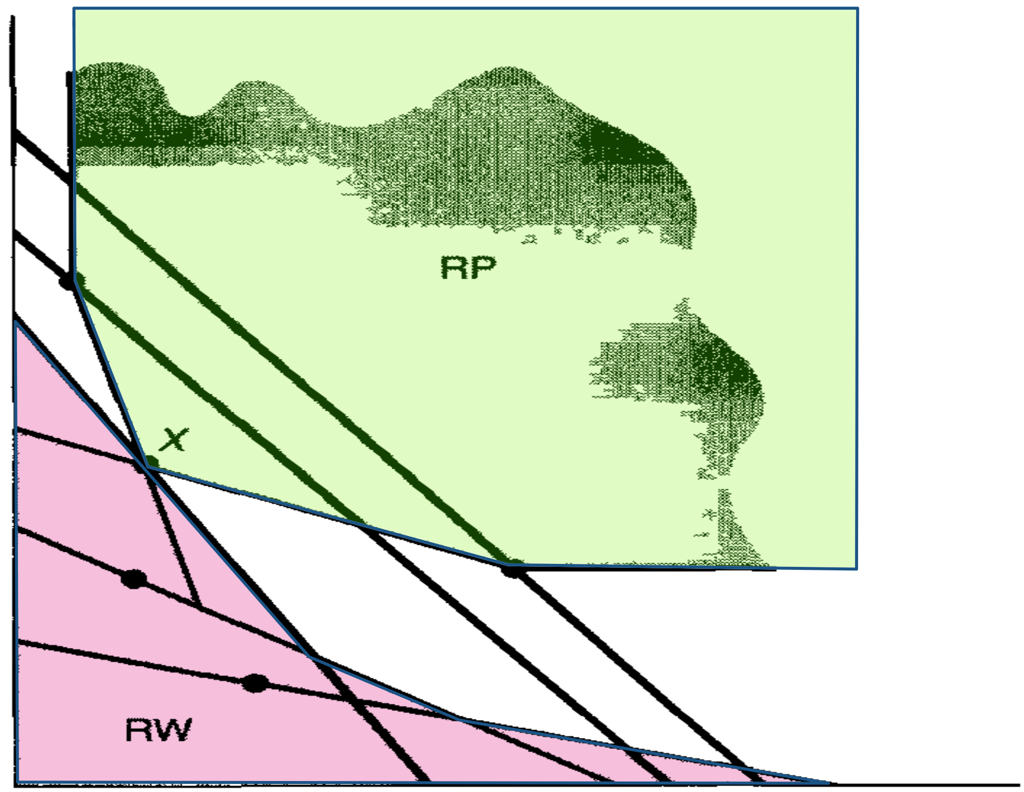
- Consider a previous observation x^1 at prices p^1 (income $w = \sum_i p_i^1 x_i^1$)
  1) What is the set of consumption bundles preferred to a new bundle x_0?
     = Green area ("revealed preferred") on the next graph
  2) What is the set of consumption bundles to which x_0 is preferred?
     = Red area ("revealed worse") on the next graph

- Indifference curve going through x_0 must lie between RP and RW areas
  - If x_0 is within the budget set when x^1 is chosen at prices p^1, bundle x^1 lies on a higher indifference curve
  - Assuming convexity, observations on segment between x_0 and x_1 are preferred to x_0, while observations South-West of that segment are worse
  - Assuming monotonicity, observations at North-East of x_0 are preferred to x_0, observations at South-West of x_0 are worse.

With only one observation previously observed:



With several observation previously observed:



Application 4: Recoverability

More on the topic:

- Recoverability with homothetic preferences:

  Strong bounds can be applied, given that only one indifference curve is sufficient to recover preferences!

- Recoverability with non-homothetic preferences:

  see Blundell et al (2003 and 2008), combining WARP with non-parametric estimates of Engel curves

Application 5: Laspeyres vs. Paasche

Definitions:

- Weighted average of changes in prices (resp. quantities)
- Weights for Laspeyres index: initial consumption (resp. prices)

$$Q^{Laspeyres} = \frac{p_x X' + p_y Y'}{p_x X + p_y Y}$$

$$P^{Laspeyres} = \frac{p'_x X + p'_y Y}{p_x X + p_y Y}$$

- Weights for Paasche (more frequently used): new consumption

$$Q^{Paasche} = \frac{p'_x X' + p'_y Y'}{p'_x X + p'_y Y}$$

$$P^{Paasche} = \frac{p'_x X' + p'_y Y'}{p_x X' + p_y Y'}$$

CPI: $\approx$ Paasche price index

Application 5: Laspeyres vs. Paasche

Can we use these indexes to make welfare statements?

Using quantity indexes:

- If $Q^{Laspeyres} = \frac{p_x X' + p_y Y'}{p_x X + p_y Y} < 1$ then $p_x X' + p_y Y' < p_x X + p_y Y$ and WARP imply that consumers are worse off now.
- If $Q^{Paasche} = \frac{p'_x X' + p'_y Y'}{p'_x X + p'_y Y} > 1$ then $p'_x X' + p'_y Y' > p'_x X + p'_y Y$ and WARP imply that consumers are better off now.
- Ambiguous results when $Q^{Laspeyres} > 1$ or $Q^{Paasche} < 1$.

Application 5: Laspeyres vs. Paasche

Can we use these indexes to make welfare statements?

Using price indexes (denoting change in income $\frac{w'}{w} \equiv \frac{p_x'X'+p_y'Y'}{p_xX+p_yY}$):

- If $P^{Laspeyres} = \frac{p_x'X+p_y'Y}{p_xX+p_yY} < \frac{w'}{w}$ then $p_x'X + p_y'Y < p_x'X' + p_y'Y'$ and WARP imply that consumers are better off now.
- If $P^{Paasche} = \frac{p_x'X'+p_y'Y'}{p_xX'+p_yY'} > \frac{w'}{w}$ then $p_xX + p_yY > p_xX' + p_yY'$ and WARP imply that consumers are worse off now.
- Ambiguous results when $P^{Laspeyres} > \frac{w'}{w}$ or $P^{Paasche} < \frac{w'}{w}$.

Application 5: Laspeyres vs. Paasche

Other application of WARP

- Comparing price indexes, we can show (exercise!):

$$P^{Paasche} < P^{Laspeyres}$$

  (assuming normal good, see lecture notes 03)

- Other price indexes: see other handout.

Application 6: Consequences of taxation

Which tax is worse?

1. Good X is taxed at a rate t (e.g. non-uniform sales tax) such that its new price is $p'_x = p_x + t$.
2. Income is taxed with a lump-sum tax L such that:

$$L = t.X^T$$

(where X^T denotes the new level of consumption with tax t).

While both taxes are equal in $, under which tax is the consumer worse off?

Application 6: Consequences of taxation

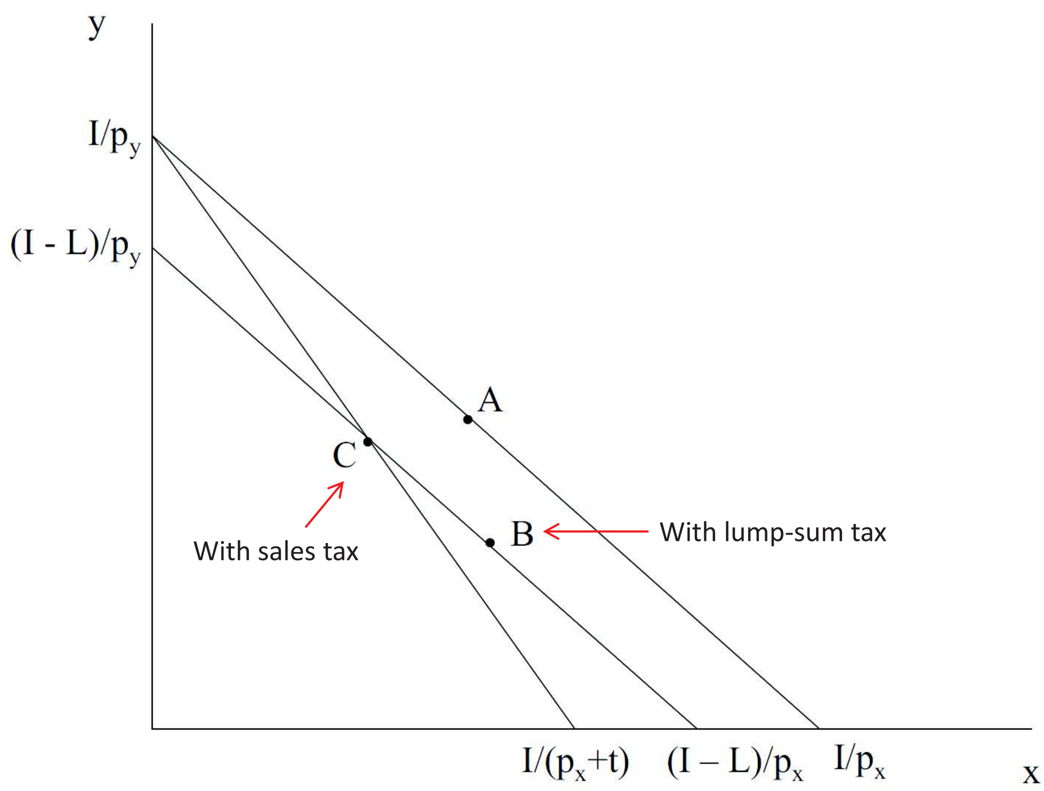
Let's (X, Y) denotes the consumption bundle before tax, (X^L, Y^L) with the lump-sum tax, and (X^T, Y^T) with the sales tax.

- Initial budget constraint implies: $p_x X + p_y Y = I$
- Lump-sum tax implies: $p_x X^L + p_y Y^L = I - L$
- Sales tax implies: $(p_x + t)X^T + p_y Y^T = I$
- Combining these three equations together with $t.X^T = L$, we obtain that (X^T, Y^T) is also on the budget line after the lump-sum tax with undistorted prices (p_x, p_y):

$$p_x X^L + p_y Y^L = p_x X^T + p_y Y^T$$

$\Rightarrow$ WARP implies that consumers are better off with lump-sum tax

Application 6: Consequences of taxation



Application 6: Consequences of taxation

Why is the lump-sum tax L better than the sales tax t?

- Because of WARP, no need for strong assumptions on preferences
- Sales tax distorts optimal consumption baskets:

$$\frac{U_x}{U_y} = \frac{p_x + t}{p_y} \neq \frac{p_x}{p_y}$$

- However, quantifying the distortion requires assumptions on the form of Utility function (we'll see that later).

Application 7: Welfare gains from trade

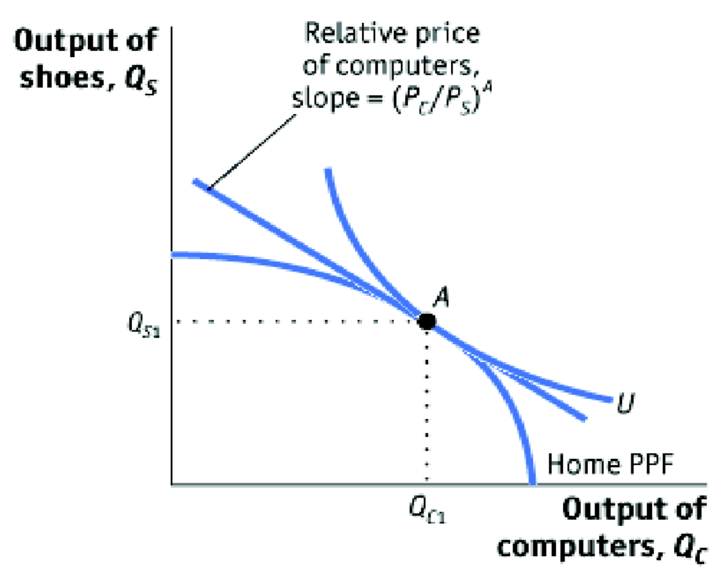
Assumption: Autarky vs. Trade:

- Production of goods C and S within the PPF ("Production Possibility Frontier"), assumed to be linear or concave
- Constant returns to scale, perfect competition, etc.
  Implies that relative prices are tangent to PPF at equilibrium
- In Autarky, relative prices are such that production equals consumption
- With trade, production bundle differs from consumption bundle, but trade is balanced

Do we always gain from trade?

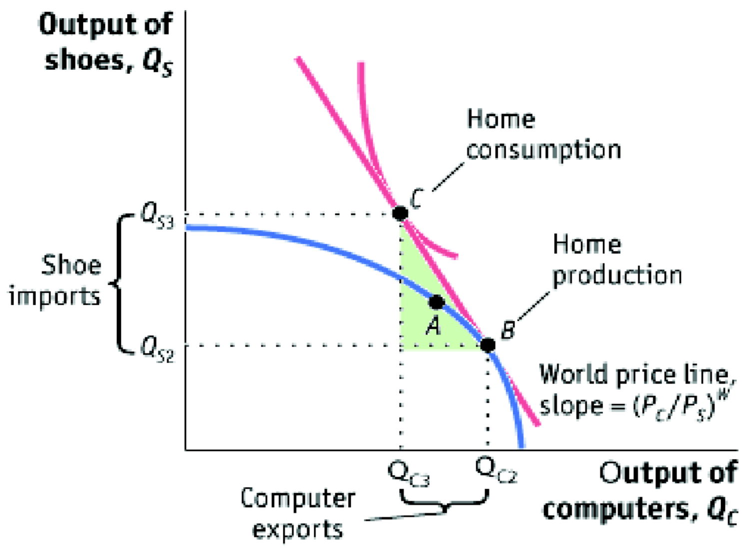
Application 7: Welfare gains from trade

Autarky equilibrium: Equilibrium consumption must be on PPF, PPF and Indifference curve must be tangent, slope is equal to relative price



Application 7: Welfare gains from trade

Trade equilibrium: PPF and Indifference curve must be tangent to budget line, slope of budget line equal to new relative price (differs from Autarky, in general)



Application 7: Welfare gains from trade

- With trade, budget line goes through production point, and its slope equals the relative price.
- Production is at equilibrium if budget line is tangent to the PPF
- From concavity of PPF (see lecture notes 4), all other points of the PPF (including Autarky equilibrium) are within the budget set

$\Rightarrow$ WARP implies that utility has to improve with Trade relative to Autarky

Note: gains from trade depend on price change (terms of trade effect) and concavity of PPF (scope of specialization)

WARP for aggregate demand?

Does WARP hold on aggregate if individual demand satisfies WARP?

- WARP obviously holds if preferences take the Gorman form
- Ok for heterogeneous but homothetic preferences
  (Note: heterogeneous homothetic prefs are not Gorman)
- Can work with some specific distributions of wealth:
  e.g. if wealth is uniformly distributed on $[0, \bar{w}]$
- Ok if satisfies "Uncompensated Law of Demand" (implies WARP):

$$(p' - p).[x_i(p', w_i) - x_i(p, w_i)] \leq 0$$

- But, unfortunately, the answer is "NO" in general...

Failure of WARP for aggregate demand

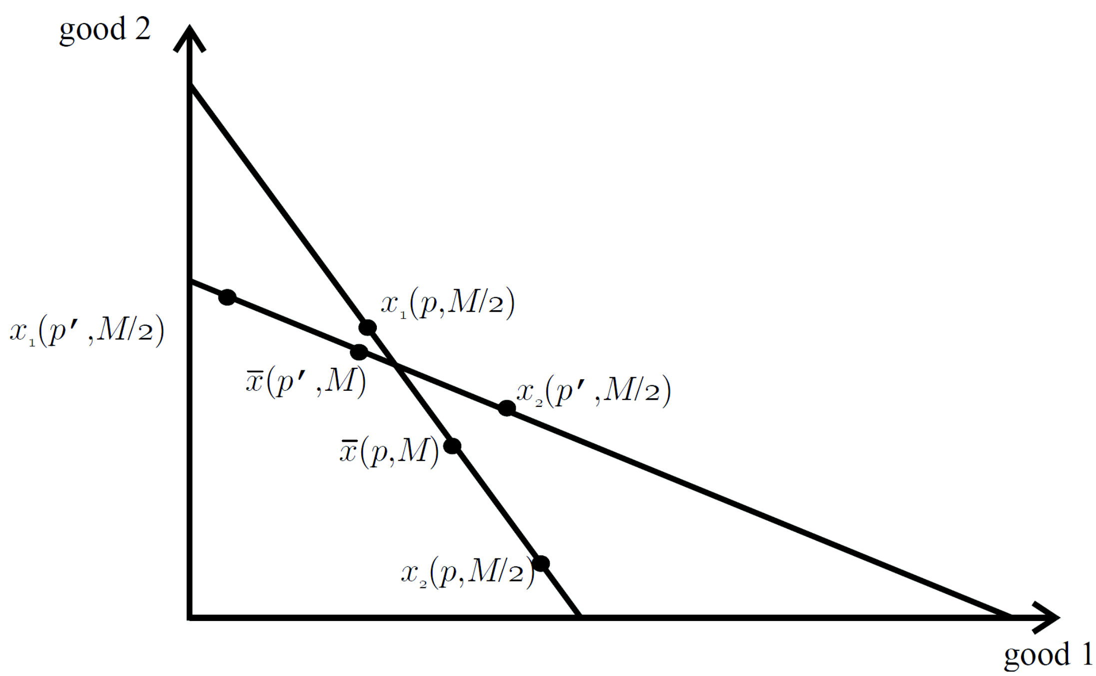


Figure 2: Failure of aggregate demand to satisfy WARP